X-ray diffraction from CuPt-ordered III-V ternary semiconductor alloy films

J. H. Li,1,2 J. Kulik,3 V. Holy, Z. Zhong, S. C. Moss, Y. Zhang, S. P. Ahrenkiel,4 A. Mascarenhas,4 and Jianming Bai5

1Physics Department, University of Houston, Houston, Texas 77204-5506
2Texas Center for Superconductivity, University of Houston, Houston, Texas 77204
3Department of Solid State Physics, Faculty of Science, Masaryk University, Kotlarska 2, 61137 Brno, Czech Republic
4National Renewable Energy Laboratory, Golden, Colorado 80401
5Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

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A model has been developed to describe x-ray scattering from CuPt-type ordered III-V ternary semiconductor alloys. The model takes into account the size distribution of the two different laminae-shaped variants, the random distribution of antiphase domain boundaries in each variant, and the atomic displacements due to the bond-length difference between the two constitutive binary materials. A synchrotron x-ray source was employed to measure the weak-ordering reflections from CuPt-ordered Ga0.5In0.5P and Al0.5In0.5As samples. By comparing the experimental results and the model calculations, structure information, including the average number of atomic layers in the laminae of each variant, the average antiphase domain size, and the average order parameter in each variant, were obtained. Results from single-variant films and poorly ordered films are also discussed.

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I. INTRODUCTION

III-V semiconductor ternary alloys $A_xB_{1-x}C$ grown on (001)-oriented substrates are known to exhibit CuPt-B-type ordering with one or two ordering variants, i.e., the [111] variant and the [111] variant, depending on the growth conditions.1–5 The properties of CuPt-ordered III-V alloys have been extensively studied in the past decade. Many interesting phenomena, such as band-gap reduction,6 valence-band splitting,7 optical anisotropy,8 conductivity anisotropy,9 second-harmonic generation10 etc., have been explored by many investigators. It is also known that two-variant ordered films have very different electronic and optical properties from those of one-variant ordered films.11 These studies have revealed that the physical properties of the ordered films depend strongly on the structural details of the material. Extensive transmission electron diffraction (TED) and microscopy (TEM) studies have successfully depicted most of the intrinsic features of the ordered phase, as well as the complex morphologies within the ordered film over a large range of length scales.12 However, most of these results remain qualitative or, even, inferential. Extended x-ray-absorption fine-structure analyses give quantitative information about the local atomic structure.13,14 These results indicate that, except for nearest neighbors, which are subject to a microscopic strain, the virtual-crystal approximation can be applied to the alloy crystals. Recently, attempts were published to model the electron-diffraction data on a microscopic scale by employing rather simplified model structures; the results, while informative from a qualitative point of view, are, nonetheless, unsatisfactory quantitatively.15,16

X-ray diffraction has long been used for a quantitative study of ordering in bulk alloys.17 However, it has not really been applied to compound semiconductor films except for the recent work of Forrest and co-workers.18,19 This is mainly because the ordering reflections from the film are weak, and the actual ordered structures are complicated, particularly when two ordering variants coexist. In this paper, we show, by using intense synchrotron x rays and developing a proper structural model that considers both the coexistence of two ordering variants and their complex interplay with randomly distributed antiphase domain boundaries, many interesting diffraction phenomena can be understood quantitatively. Effects of A-C and B-C bond-length differences on the diffraction profiles are also considered in our model, which, as will be demonstrated in the paper, has a strong impact on the determination of the order parameter.

This paper is organized as follows. After the introduction, results of TED and TEM on the CuPt-ordered $A_xB_{1-x}C$ materials will be briefly reviewed in Sec. II. Experimental procedures are given in Sec. III. In Sec. IV, a structural model and a kinematical x-ray-diffraction theory are described. The effects of atomic displacements due to the bond-length difference between A-C and B-C bonds on the diffraction data are discussed in Sec. V. Results of the synchrotron x-ray diffraction from our GaInP$_2$ and AlInAs$_2$ films and their interpretations, are presented in Sec. VI. In Sec. VII, we discuss the problems in determining the order parameter from the ordering reflections. Conclusions of this study are given in Sec. VIII.

II. RESULTS FROM TED AND TEM STUDIES

A typical TED pattern from an ordered $A_xB_{1-x}C$ film with double variants is shown in Fig. 1. The ordering reflections are elongated along directions $\sim 10^\circ$ away from the [001] direction, forming a characteristic wavy pattern. The cause of this unusual diffraction phenomenon is believed to be the complex structure of the ordered film, which allows the two variants to interlock with the antiphase domain boundaries to form plate-like domains slightly inclined with respect to the [001] direction.12,20,21 In Fig. 2, atomic images taken from a single-phase, double-variant domain and a single-phase, single-variant domain are shown. In the case of double variants, a laminar structure with alternating variants
along the [001] direction is clearly observed. A detailed description of the laminar structure of the two variants and their interlock with the antiphase domain boundaries can be found in Ref. 12, and references therein.

III. SAMPLE PREPARATION AND X-RAY DIFFRACTION

Samples studied in this work are nominally Ga_{0.5}In_{0.5}P grown on GaAs(001) substrates and Al_{0.5}In_{0.5}As grown on InP(001) substrates by metal-organic chemical vapor deposition. The substrates are tilted by about 6° toward either the $\{111\}_A$ or $\{111\}_B$ direction. X-ray diffraction was performed on beamline X14A of the National Synchrotron Light Source at the Brookhaven National Laboratory, with an energy of 8.0478 keV. A Si(111) monochromator set slightly away from the Bragg position, and a pulse height analyzer, were used to remove the higher-order harmonics ($\lambda_n = \lambda/n$). This consideration is particularly important for our experiments because the second-order harmonic would appear exactly at the positions where the ordering reflection would occur [although, in principle, there is no Si(222) reflection, a weak one can be actually measured due to a loss in inversion symmetry of the binding electrons]. A proportional counter was used to record the diffracted x rays. A noncoplanar diffraction geometry (i.e., the plane of diffraction defined by the incident and the diffracted beams is not perpendicular to the sample surface) was introduced in our experiments by using the $\chi$ angle on a standard four-circle diffractometer, so that the reflections, such as ($\bar{1}11$) and ($3/2 3/2 3/2$), which are usually not accessible in a conventional coplanar diffraction geometry, can be reached. This noncoplanar geometry is schematically shown in Fig. 3. The incident slits, 0.1 (in plane) $\times$ 0.5 mm$^2$ (out of plane), and the receiving slits, 0.5 (in plane) $\times$ 0.5 mm$^2$ (out of plane), give a resolution of the diffractometer of about 0.006 in dimensionless reciprocal-lattice units. A post-measurement intensity correction, considering the effective size of the footprint of the incident x-ray beam on the sample surface, is necessary if the scan range is large.

IV. STRUCTURAL MODEL AND DIFFRACTION THEORY

The two possible CuPt ordering variants are schematically shown in Figs. 4(a) and 4(b), respectively. In the text of the paper, we will call the ($\bar{1}11$) and (111) variants variants I and II, respectively. In the diagram of variant I, an antiphase

along the [001] direction is clearly observed. A detailed description of the laminar structure of the two variants and their interlock with the antiphase domain boundaries can be found in Ref. 12, and references therein.
domain boundary perpendicular to the \([\bar{1}10]\) direction is also shown. Such boundaries will inevitably exist in the film, because the initial occupation of a group-III lattice site by atom \(A\) or \(B\) is equally possible, and the growth actually occurs simultaneously everywhere over the substrate surface. In Figs. 4(c) and 4(d), two examples of a laminar structure with alternating variants are shown. The difference between Figs. 4(c) and 4(d) lies in the thickness of the variant-II lamina, and the effect this has on the relationship between the neighboring variant-I laminae. When the two variant I laminae are separated by a lamina of variant II containing an even number of group-III atomic layers [as in Fig. 4(c)], the variant-I laminae are in phase with each other. Conversely, if the intervening variant II lamina is an odd number of group-III atomic layers in thickness [e.g., Fig. 4(d)], the variant-I laminae will then be out of phase. In a real case, the number of group-III atomic layers in a lamina of either variant may be random.

Based on these considerations, a realistic structural model must consist of statistically distributed lateral antiphase domains and a varying phase relationship in the growth direction, as shown in Fig. 5, where the antiphase domains in the two variants are considered to be independent from each other. The number of atomic layers in each lamina is random.

To describe the diffraction theory in steps, let us first neglect the antiphase domain boundaries in the \([\bar{1}10]\) direction, the atomic layers in the \([\bar{1}10]\) direction are assumed to be infinite. The basic structural units that constitute the ordered phase are a layer of group-III atoms and a layer of group-V atoms. In order to obtain a structure of variant I, we have to shift the next group-III atomic layer with respect to the previous one by a vector \((-a/2,0,a/2)\) in the crystallographic coordinate system. The phase shift in the structure factor of this layer caused by such an operation is

\[
\alpha = e^{\pi i (q_1 + q_3)},
\]

where \((q_1, q_2, q_3)\) are the coordinates of the scattering vector \(Q\):

\[
Q = \frac{2\pi}{a} (q_1, q_2, q_3).
\]

Similarly, the phase shift between the structure factors of neighboring atomic layers in variant II is

\[
\beta = e^{\pi i (-q_1 + q_3)}.
\]

Let us assume a vertical sequence of \(N\) pairs of laminae of variants I and II. The \(j\)th pair of laminae consists of \(n_{Aj}\) and \(n_{ Bj}\) atomic layers of group-III atoms in variants I and II, respectively. These numbers are random. If we denote the structure factors of infinite layers in variants I and II as \(F_{0A,B}(Q)\), the structure factor of the whole stack of laminae is then given by

\[
F(Q) = \sum_{j=1}^{N} \left[ F_{0A}(Q) \alpha^{n_{Aj}-1} + F_{0B}(Q) \beta^{n_{Bj}-1} \right] \times (\alpha \beta)^{-1} \prod_{k=1}^{j-1} \alpha^{n_{Ak}-1} \beta^{n_{Bk}-1}.
\]

The x-ray intensity scattered by such a structure can thus be written in the form

\[
I(Q) \propto \left| \langle F(Q) \rangle \right|^2 = \sum_{j=1}^{N} \left[ \left( F_{0A}(Q) \alpha^{n_{Aj}-1} \right)^2 + F_{0B}(Q) \beta^{n_{Bj}-1} \left( F_{0A}(Q) \alpha^{n_{Aj}-1} \right) \right] + 2 \text{Re} \left( \sum_{j=2}^{N} \left( \alpha \beta \right)^{-j} \left( F_{0A}(Q) \alpha^{n_{Aj}-1} \right)^2 + F_{0B}(Q) \beta^{n_{Bj}-1} \left( F_{0A}(Q) \alpha^{n_{Aj}-1} \right) \right) \times \prod_{k=1}^{j-1} \left( \alpha^{n_{Ak}-1} \beta^{n_{Bk}-1} \right) \right]
\]

Here \(\langle \rangle\) indicates an average over all possibilities of \(n_{Aj}\) and \(n_{Bj}\). We denote \(\chi_{a} = \langle \alpha \rangle\), and \(\chi_{B} = \langle \beta \rangle\) and Eq. (4) can be written as:

\[
I(Q) \propto 2N \left[ |F_{0A}(Q)|^2 \frac{1 - \text{Re} \left( \chi_{a} \right)}{|1 - \alpha|^2} + |F_{0B}(Q)|^2 \frac{1 - \text{Re} \left( \chi_{B} \right)}{|1 - \beta|^2} \right]
\]

\[
- \text{Re} \left( F_{0A}(Q) F_{0B}^* (Q) \frac{1 - \chi_{a}^*}{1 - \alpha} \frac{1 - \chi_{B}^*}{1 - \beta} \right)
\]

\[
+ 2 \text{Re} \left( F_{0A}(Q) \alpha \chi_{B} \frac{1 - \chi_{a}}{1 - \alpha}^2 \right)
\]

\[
+ |F_{0B}(Q)|^2 \beta \chi_{a} \frac{1 - \chi_{B}}{1 - \beta}^2 + |F_{0A}(Q) F_{0B}(Q) \beta \chi_{a} \chi_{B} \right]
\]

\[
+ F_{0A}(Q) F_{0B}(Q) \alpha \frac{1 - \chi_{a}}{1 - \alpha} \frac{1 - \chi_{B}}{1 - \beta} \chi_{a} \chi_{B}
\]

\[
\chi_{a} \chi_{B} \left( N - 1 - \frac{1 - (\chi_{a} \chi_{B})^{N-1}}{1 - \chi_{a} \chi_{B}} \right).
\]
If we assume that the numbers \( n_{A_j} \) and \( n_{B_j} \) follow a Poisson distribution with mean values \( \langle n_{A_j} \rangle = n_a \) and \( \langle n_{B_j} \rangle = n_b \), we obtain

\[
\chi_x = e^{-u_{j}(1-\gamma)}, \quad \gamma = \alpha, \beta.
\] (6)

Let us now take into account the lateral antiphase domains in each lamina. In this case, each lamina contains alternating segments out of phase with respect to each other. For the sake of simplicity, we assume that the lateral sequence of these segments is statistically not correlated to the vertical sequence of the two variants. Then the existence of the lateral antiphase domains affects only the terms \( |F_{0a}(Q)|^2 \), \( |F_{0b}(Q)|^2 \), and \( F_{0a}(Q)F_{0b}(Q) \) in Eq. (5). Now these terms must be replaced by their averages, taking into account a sequence of randomly distributed lateral segments. If we further assume that the lateral structures of different laminae are not correlated, then we simply need to make the following replacements in Eq. (5):

\[
|F_{0a}(Q)|^2 \to \langle |F_{0a}(Q)|^2 \rangle, \quad |F_{0b}(Q)|^2 \to \langle |F_{0b}(Q)|^2 \rangle, \quad F_{0a}(Q)F_{0b}(Q) \to \langle F_{0a}(Q) \rangle \langle F_{0b}(Q) \rangle^*.
\] (7)

The structure factor of the \( p \)th lateral segment of a layer is shifted by a phase factor \( \phi \) with respect to the segment \( p-1 \),

\[
\phi = e^{i\pi(-q_1+q_2)} = e^{-2\pi i q_1},
\]

if we assume \( q_1 = q_2 \) in the \([1\overline{1}0]\) scattering plane. The structure factor of such a sequence of segments is

\[
F_{0S}(Q) = \sum_{p=1}^{M} F_{0S}^{(p)} \phi_{x}^{p-1}, \quad S = A, B,
\] (8)

where

\[
F_{0S}^{(p)} = F_{cS} \phi_{x}^{2m_p - 1} = \phi_{x}^{2m_p - 1} \prod_{k=1}^{p-1} \phi_{x}^{2m_k}, \quad S = A, B
\] (9)

is the structure factor of the \( p \)th segment of variant-I or -II lamina containing \( m_p \) unit cells, \( M \) is the number of segments in one lamina, and \( F_{cS} \) is the structure factor of one unit cell as defined in Fig. 4. We therefore have

\[
\langle |F_{0S}(Q)|^2 \rangle = |F_{cS}(Q)|^2 \sum_{p=1}^{M} \left| \left( \frac{\phi_{x}^{2m_p - 1}}{\phi_{x}^{2} - 1} \right)^2 \right| ^2
\]

\[
+ 2 \text{Re} \left( \sum_{p=2}^{M} \sum_{k=1}^{p-1} \phi_{x}^{2m_p - 1} \phi_{x}^{2m_k - 1} \right) \prod_{r=k}^{p-1} \phi_{x}^{2m_r} \right], \quad S = A, B.
\] (10)

After some manipulation, we obtain

\[
\langle |F_{0S}(Q)|^2 \rangle = |F_{cS}(Q)|^2 \left[ \frac{M(1 - \text{Re}(\chi_\|))}{\phi_{x}^{2} - 1} \right]^2
\]

\[
- \text{Re} \left[ \frac{\phi_{x}^{2} \chi_{\|} (1 - \chi_\|)^2}{1 - \phi_{x}^{2} \chi_{\|}} \right]
\]

\[
\times \left( M(1 - \frac{(\phi_{x}^{2} \chi_{\|})^M - 1}{\phi_{x}^{2} \chi_{\|} - 1}) \right), \quad S = A, B.
\] (11)

where

\[
\chi_\| = \langle \phi_{x}^{2m} \rangle
\] (12)

is the averaged phase shift over all random numbers \( m_p \) of the unit cells in a segment. In a similar way, we obtain

\[
\langle F_{0S}(Q) \rangle = F_{cS}(Q) \frac{1 - \chi_\| - (\phi_{x}^{2} \chi_{\|})^M}{1 - \phi_{x}^{2} - \phi_{x}^{2} \chi_{\|}}, \quad S = A, B.
\] (13)

The averaged phase shift \( \chi_\| \) can be calculated readily if we assume that the random numbers of \( m_p \) follow a Poisson distribution with an average \( \langle m_p \rangle = m \).

\[
\chi_\| = e^{-m}(1 - \phi_{x}^{2}).
\] (14)

Different configurations of antiphase domain boundaries can now be achieved by different combinations of the parameters \( n_a \), \( n_b \), and \( m \).

V. EFFECT OF THE ATOMIC DISPLACEMENTS

Bulk semiconductor alloys \( A_1B_1-C \) are known to obey Vegard’s law quite accurately: the overall lattice constant is the average of the bulk \( AC \) and \( BC \) lattice constants, weighted by mole fraction. If we imagine the alloy to be a virtual crystal (VC), in that each atom sits on geometrically precise zinc-blende lattice sites, then its lattice constant can be expressed as

\[
a_{ABC} = a_{AC} + (1 - x)a_{BC}.
\]

Indeed, measurements\(^{13,14}\) show that the second-nearest-neighbor distances between group-III atoms (or between group-V atoms) in the lattice are very nearly those \(-a_{ABC}/\sqrt{2} = \sqrt{3}a_{ABC}/4\), expected from the VC approximation.

In contrast, however, first-nearest-neighbor distances between group-III and -V atoms deviate significantly from \( \sqrt{3}a_{ABC}/4\), expected from the VC approximation. It is always that the \( A-C \) (or \( B-C \)) bond length is shorter (or longer) than the VC bonds. Therefore, one can imagine that, instead of occupying the VC lattice sites, the atoms will sit in positions slightly deviated from the VC sites, resulting in a microscopic strain in the alloy. Such deviations or microscopic strain have been treated in great detail in the past 20 years, employing mainly what are known as valence-force-field models,\(^{22-24}\) in which the energies of individual bonds and bond angles are considered to be independent of each other. By minimizing the total energy of the typical tetrahedral bond structure in diamond-type semiconductors, the actual bond lengths, or the first-nearest-neighbor distances, and the displacements of the atoms from their VC sites, can be de-
assuming that the term in Eq. (15) over all the atomic sites displacement of the atom at \( r_i \) is composed of two components, the mean atomic form factor \( \bar{f}_i \) of a disordered crystal at lattice site \( r_i \) in the VC approximation, and the deviation of the atomic form factor from \( \bar{f}_i \) due to the atomic ordering, \( f(r_i) \). \( \delta_i \) is the displacement of the atom at \( r_i \). The summation in Eq. (15) is over all the atomic sites \( r_i \) in the unit cell. The exponential term in Eq. (15) can be expanded to

\[ e^{2\pi iQ \cdot r_i(1 + 2\pi iQ \cdot \delta_i)} \]

assuming that \( \delta_i \) is much smaller than the VC bond length. To obtain the x-ray intensity, we have to calculate the average of \( |F_{cS}|^2 \) over all possibilities of atom arrangements due to ordering, and all the possibilities of the atomic displacements. If we assume that the atomic form factors for the atoms at different lattice sites and the displacements of the atoms at these sites are not correlated, and further assume that \( f_i(r_i) \) and \( \delta_i \) at the same site are independent variables (note that the displacement of an atom at \( r_i \) depends mainly on its first-nearest neighbors, and is rather an intrinsic property of the ABC\(_{2}\)-type semiconductor alloys than a result of atomic ordering), then we have

\[
|F_{cS}|^2 = \left( \sum_i \sum_j |f_i(r_i)|f_j^*(r_j) e^{2\pi iQ \cdot (r_i + \delta_i)} e^{-2\pi iQ \cdot (r_j - \delta_j)} \right)^2
\]

\[
= \left( \sum_i \sum_j \langle f_i(r_i) \rangle \langle f_j(r_j) \rangle^* e^{2\pi iQ \cdot (r_i - r_j)} \right)^2
\]

\[
\times (1 + 2\pi iQ \cdot \delta_j) \times (1 + 2\pi iQ \cdot \delta_i)
\]

\[
= \sum_i \sum_j \langle f_i(r_i) \rangle \langle f_j(r_j) \rangle^* e^{2\pi iQ \cdot (r_i + \delta_i)} e^{-2\pi iQ \cdot (r_i + \delta_i)}
\]

\[
\times (1 + 2\pi iQ \cdot \delta_j) \times (1 + 2\pi iQ \cdot \delta_i)
\]

\[
= \sum_i \sum_j \langle f_i(r_i) \rangle \langle f_j(r_j) \rangle^* e^{2\pi iQ \cdot (r_i + \delta_i)} e^{-2\pi iQ \cdot (r_j - \delta_i)}
\]

\[
\times (1 + 2\pi iQ \cdot \delta_j) \times (1 + 2\pi iQ \cdot \delta_i)
\]

Now Eq. (15) can be rewritten as

\[
F_{cS} = \sum_i \langle f_i(r_i) \rangle e^{2\pi iQ \cdot (r_i + \delta_i)}
\]

It can also be written in the form

\[
F_{cS} = \sum_i \{ \bar{f}_i + (f(r_i)) + 2\pi iQ \cdot \delta_i \} e^{2\pi iQ \cdot r_i}
\]

\[
= F_0 + F_s + F_d,
\]

where

\[
F_0 = \sum_i \bar{f}_i e^{2\pi iQ \cdot r_i}
\]

\[
F_s = \sum_i \langle f(r_i) \rangle e^{2\pi iQ \cdot r_i}
\]

\[
F_d = 2\pi i \sum_i Q \cdot \delta_i \{ \bar{f}_i + (f(r_i)) \} e^{2\pi iQ \cdot r_i}
\]

are the contributions to the total structure factor from a perfect VC, from atomic ordering and from atomic displacements, respectively. The x-ray intensity is calculated by

\[
|F_{cS}|^2 = (F_0 + F_s + F_d)^2 = (F_0^* + F_s^* + F_d^*)^2
\]

\[
= F_0 F_0^* + F_s F_s^* + 2 \text{Re}(F_0 F_s^*) + 2 \text{Re}(F_d F_0^*)
\]

\[
+ 2 \text{Re}(F_d F_s^*) + F_d F_d^*.
\]

The first term in Eq. (22) gives the fundamental reflections. The second term is caused by the atomic ordering. The third term is zero because \( \Sigma f(r_i) = 0 \). The fourth and fifth terms result in diffuse intensities around the fundamental and ordering reflections, respectively. The last term, though much weaker than the other terms, represents the contribution purely due to the atomic displacements. In our case, because the displacements are also ordered, as we will show in the following discussion, the atomic displacements bring in an additional modulation to the ordering reflections. Therefore, the intensity of an ordering reflection from a single variant domain is

\[
I_{\text{order}} \propto |F_s|^2 + 2 \text{Re}(F_d F_s^*) + |F_d|^2.
\]

In addition to the bond-length-difference-induced permanent atom displacements, the thermal vibration of the atoms causes a random displacement. The effect of this thermal vibration on the x-ray structure factor is considered via Debye-Waller factors. In this paper, the Debye-Waller factors were calculated from the estimated Debye temperatures using the Debye model, resulting in \( B_{\text{InP}} = 0.2799 \), \( B_{\text{GaP}} = 0.3001 \), \( B_{\text{InAs}} = 0.206 \), and \( B_{\text{AlAs}} = 0.283 \).

Before calculating the average atomic form factor and the average atom displacements, let us introduce the well-accepted Bragg-Williams long-range order parameter. If we define \( \gamma_A(\gamma_B) \) and \( \omega_A(\omega_B) \) as the fraction of A sites (B sites) occupied by the right and wrong atoms, respectively, we then have

\[
\gamma_A(\gamma_B) = \frac{1}{2} \left( 1 + \text{Re} \left( \frac{\omega_A(\omega_B)}{\omega_A(\omega_B) + 1} \right) \left( 1 + \text{Re} \left( \frac{\omega_A(\omega_B)}{\omega_A(\omega_B) + 1} \right) \right) \right)
\]

\[
\omega_A(\omega_B) = \frac{1}{2} \left( 1 - \text{Re} \left( \frac{\gamma_A(\gamma_B)}{\gamma_A(\gamma_B) + 1} \right) \left( 1 - \text{Re} \left( \frac{\gamma_A(\gamma_B)}{\gamma_A(\gamma_B) + 1} \right) \right) \right)
\]
Wrong atoms at group-III lattice sites are Bearing this in mind, the possibilities of finding the right and For simplicity, here we will discuss only the case in which more complicated. In an (110) lattice planes. Consequently, the displacements of the atoms have different values. The direction of the average displacements are perpendicular to the ordered {111} lattice planes. In Sec. VI, we will show how this displacement influences the diffraction profiles.

VI. RESULTS AND DISCUSSION

In this section, we present and discuss the experimental results and theoretical fitting results for several Ga0.5In0.5P and Al0.5In0.5As films, which are quite representative of the CuPt-type ordered III-V semiconductor alloys. Samples MA776 and MA912 are Ga0.5In0.5P films grown on GaAs (001)-6°A substrates. Sample K782 is a Ga0.5In0.5P film grown on an InP(001)-6°B substrate. Sample R286 is an Al0.5In0.5As film grown on an exactly oriented InP(001) substrate. Growth conditions of the Ga0.5In0.5P films and the Al0.5In0.5As film can be found in Refs. 11 and 18, respectively. Figure 7 is a schematic diagram of the [110]-zone reciprocal-lattice plane within which most of the scans in this work were made. It is well known that a film grown on a substrates miscut toward the (111)-B direction would result in single variant in the film.1-6 In such a case, only one set of the ordering reflections were observed by TED. No wavy or tilted ordering peaks were observed in such films. Our x-ray measurements on sample K782 indeed confirm that only one variant appears in this film grown on a 6°B substrate. The measured reciprocal area map of this sample around (312,312,52) ordering reflection is shown in Fig. 8(a). The intensities of the fundamental reflections and their tails in the measured area have been subtracted from the map in order to highlight the rather weak ordering peak. This procedure has been performed on all the data discussed below. The peak in Fig. 8(a) does indeed look close to a circle as already noticed by TED observations. Figure 8(b) is the calculated (312,312,52) reflection. Figure 8(c) shows the cross section.
at $Q_{[001]}=2.5$. The best agreement between the calculated and measured data was achieved by using a domain size of about 500 crystallographic unit cells. Here it is worth pointing out that the intensities of the ordering reflections have been modulated by the effect of atomic displacements as discussed in Sec. V. As an example, Fig. 9 shows the calculated x-ray radial scans from reciprocal lattice points ($\bar{1}11$) to (333) for a perfectly ordered large single-variant domain. The central peak in each panel is the fundamental (222) reflection. Figure 9(a) is the result without taking into account the atomic displacements. Naturally, the (322) reflection is stronger than the higher-order (525) reflection, as the atomic form factor decreases with increasing scattering vector. In Fig. 9(b), scattering due to the atomic displacements is shown, which gives a modulation to the above profile. In Fig. 9(c), the total intensity profile considering the atom displacements is shown. The (525) reflection here is considerably stronger than the (322) reflection.

Figure 10(a) is an experimental x-ray area scan in reciprocal space taken from sample MA776 around the ordering reflections ($\bar{T}2,7/2,5/2$) and ($\bar{T}2,7/2,7/2$). It is clear that both ordering peaks are inclined away from the [001] direction by an angle of about 10°. This feature is quite similar to the typical TED data shown in Fig. 1. It is easy to recognize that the ($\bar{T}2,7/2,7/2$) and ($\bar{T}2,7/2,5/2$) reflections are contributions of the ordering variants I and II, respectively. We also note that the reflection ($\bar{T}2,7/2,7/2$) is much stronger than the reflection ($\bar{T}2,7/2,5/2$). However, we must not conclude from this feature that the fraction of the variant I in this sample is larger than that of the variant II. Figure 10(b) gives the calculated intensity distribution by using Eq. (5). The parameters used for the calculation are listed in Table I. Both maps agree qualitatively quite well but for a detailed com-

FIG. 8. X-ray reciprocal-space area maps taken from a single-variant film: (a) experimental, (b) calculated, and (c) cross-sectional line scans at $Q_{[001]}=2.5$.

FIG. 9. Calculated x-ray line scans from reciprocal-space lattice points ($\bar{1}11$) to (333). (a) Without taking into account the atomic displacements. (b) Modulation caused by atomic displacements. (c) Results of (a)+(b).

FIG. 10. X-ray reciprocal space area scans of sample MA776: (a) experiment; (b) calculation. The peaks centered at ($\bar{T}2,7/2,7/2$) and ($\bar{T}2,7/2,5/2$) are attributed to variants I and II, respectively.
 TABLE I. The parameters used to fit the measured data of samples MA776, MA912, and R286. \( \langle n_\alpha \rangle \), \( \langle n_\beta \rangle \), and \( \langle m \rangle \) are the average number of group-III atomic layers in the laminae of variants I and II, and the average antiphase domain size. \( \langle s_1 \rangle \) and \( \langle s_2 \rangle \) are the average order parameters of variants I and II, respectively.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \langle n_\alpha \rangle )</th>
<th>( \langle n_\beta \rangle )</th>
<th>( \langle m \rangle )</th>
<th>( \langle s_1 \rangle )</th>
<th>( \langle s_2 \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA776</td>
<td>7</td>
<td>8</td>
<td>60</td>
<td>0.35±0.05</td>
<td>0.45±0.05</td>
</tr>
<tr>
<td>MA912</td>
<td>12</td>
<td>12</td>
<td>140</td>
<td>0.50±0.05</td>
<td>0.50±0.05</td>
</tr>
<tr>
<td>R286</td>
<td>4</td>
<td>4</td>
<td>~</td>
<td>0.30±0.05</td>
<td>0.30±0.05</td>
</tr>
</tbody>
</table>

aFor ordered domains only.

The comparison of the calculation and the experiment, area maps are not suitable. In Figs. 11(a) and 11(b), two cross sections at \( Q_\parallel[001] = 2.5 \) and \( 3.5 \) are shown. We see that the theory yields a quite good fit to the experimental data. In Fig. 12, a fit, using the same parameters, to a line scan from reciprocal-lattice point \( (1\overline{1}3) \) to \( (3\overline{3}5) \) is shown, and the agreement is also good increasing our confidence in the model.

Figure 13(a) shows the experimental area map taking from sample MA912. Figure 13(b) is the calculated one using Eq. (5). The structural parameters used for this calculation are listed in Table I. To view quantitatively the fitting results, several line scans along either \( [1\overline{1}1] \) or \( [1\overline{1}1] \) directions are shown in Fig. 14, namely, scans from the reciprocal lattice point \( (1\overline{1}1) \) to \( (3\overline{3}3) \), \( (0\overline{0}2) \) to \( (2\overline{2}) \), \( (1\overline{1}3) \) to \( (3\overline{3}1) \), and \( (0\overline{0}2) \) to \( (2\overline{2}4) \). Good agreement has been obtained between the several experimental and simulated curves.

From the calculations, we note that the widths of the ordering reflections are determined by the combination of lateral antiphase domain size and the laminar thicknesses, i.e., the overall configuration of the antiphase domain boundaries. This is also true for the tilt angle of the ordering reflections. To show this point more clearly, a series of calculated area scans using a Ga\(_0.5\)In\(_0.5\)P film as the model structure are shown in Figs. 15 and 16. By varying the average thicknesses of the laminae of the two variants and the lateral domain size, we are able to obtain quite different diffraction patterns. From Fig. 15, we notice that when the antiphase domains are very small in size, each ordering reflection has split into two. The smaller the antiphase domains, the greater the separation of the two split maxima are located. In fact, such splittings of ordering reflections and shifts of intensity maxima have been observed by TED on poorly ordered films.12 (The TED pattern in Fig. 1 is an example of such a case). It is also clear that when the antiphase domains are very large, the ordering reflections become very narrow and lie almost parallel to the [001] direction. This latter finding is further confirmed by Fig. 16(d), which represents an area scan for a two-variant structure without lateral antiphase domains but with statistically distributed laminae thicknesses. In this case, the ordering reflection is elongated simply along the [001] direction, but is no longer tilted to form the wavy pattern. Therefore, dense antiphase domain boundaries are responsible for the tilted reflection peaks.

Not all of the films, however, are well ordered, by which we mean that the two ordered variants are complementary in the film. Some films grown under certain conditions may not be well ordered, in which ordered domains are embedded in the disordered matrix.15,28 In this case, TEM study has shown that the ordered domains have a finite lateral size, but, in the [001] direction, the ordering develops quite well. Overall, these ordered domains are characterized by a columnar structure.15 Based on the theory first developed by Greenholz...
and Kidron, in which the interference between the scattering from different ordered domains is neglected, we can calculate the x-ray scattering intensity from a columnar structure by keeping the same vertical structural model, as we used above for the well ordered materials. Figure 17(a) is a reciprocal-space area scan taken from sample R286. This sample was shown to exhibit short-range order from our earlier work. The general feature of this figure, particularly the inclination of the streaks passing through the \((\bar{3}31)\) positions, is quite similar to that of the well-ordered materials. The elongated streaks are clear indications of small ordering domains in the film. An interesting feature here is the presence of streaks parallel to the \([\bar{0}01]\) direction, running between the ordering reflections. The ridge of intensity between \((331)\) and \((113)\) is due to the high density of planer stacking faults. See Ref. 18 for details.

Figure 17(b) is the calculated area map using the columnar structure model and the parameters listed in Table I. It fits qualitatively quite well to Fig. 17(a), considering the shape and tilt of the ordering reflections, as well as the \([001]\) streaks between them. In fact, the quantitative fit is also good if we show cross-sectional line scans running through the ordering peaks (Fig. 18). We see from Fig. 18 that both the line shape and intensity of the calculated curves are in good agreement with their experi-

FIG. 14. X-ray line scans from one reciprocal-lattice point to another. (a) From \((111)\) to \((333)\). (b) From \((002)\) to \((\bar{2}24)\). (c) From \((1\bar{1}3)\) to \((331)\). (d) From \((002)\) to \((220)\). Solid dots and lines are measured and calculated data, respectively.

FIG. 15. A series of calculated area scans for an ordered Ga\(_{0.5}\)In\(_{0.5}\)P model structure with the parameters \(\langle n_a\rangle = \langle n_b\rangle = 8\) and \(\langle m\rangle = 5\), (b) \(\langle m\rangle = 10\), (c) \(\langle m\rangle = 20\); (d) \(\langle m\rangle = 40\), (e) \(\langle m\rangle = 60\), and (f) \(\langle m\rangle = 200\).

FIG. 16. Calculated area scans for an ordered Ga\(_{0.5}\)In\(_{0.5}\)P model structure with the structural parameters \(\langle n_a\rangle = \langle n_b\rangle = 8\), (a) \(\langle n_a\rangle = \langle n_b\rangle = 8\), and (c) \(\langle n_a\rangle = \langle n_b\rangle = 20\). (d) is calculated by using \(\langle n_a\rangle = \langle n_b\rangle = 8\), and assuming that the film is free of lateral anti-phase-domain boundaries.
However, many researchers, including us, have used this method in earlier studies.\textsuperscript{30,31} There is no doubt that this method will produce correct values for good single crystals. The almost inevitable presence of antiphase displacements. The almost inevitable presence of antiphase reflections from the substrate and film overlap one another. This is because the film is usually lattice matched to the substrate material, and the reflections are often difficult. This is because the actual structure of the film is more complicated than our model, which permits us to include static displacement diffuse scattering near the fundamental peaks (Huang scattering). In addition, the actual film may contain defects such as diffuse antiphase domain boundaries,\textsuperscript{12} stacking faults,\textsuperscript{1,4} alloy clustering\textsuperscript{15,32} etc. The interface of the neighboring laminae of the two variants may also not be as abrupt as we considered in our model. It is worth pointing out that the order parameter of sample MA776, determined from a direct comparison of the intensities of the (1/2,1/2,3/2) ordering reflection and the (004) fundamental reflection using the methods discussed in Ref. 30, is about 0.06 for both variants, taking the volume fractions of the two variants determined in Sec. VI.

Finally, we note that the precise determination of the order parameter for the complex ordered film by diffraction methods is still a challenge. Our model assumes that the laminar structure runs uniformly across the wafer, and the number of atomic layers in a single lamina follows a certain distribution. Such assumptions may not be always true. Electron diffraction may indeed be better in determining the local order parameter if one can focus the electron beam onto a single-phase, single-variant domain, and use kinematic theory (always a problem with TED).

VIII. CONCLUSIONS

On the basis of a theory of kinematic diffraction and a structure model of CuPt-type ordering in III-V compound...
semiconductors, considering the laminae structure of two alternating variants and the random distribution of antiphase domain boundaries in each variant, we have calculated the reciprocal-space distribution of the x-ray intensities scattered from CuPt-type ordered $A_B$B$_{1-x}$C materials. Atomic displacements due to the bond-length difference between $A$C and $B$C were also considered in our model. We find that, since atomic displacements are associated with atomic ordering, the average atomic displacements of alternating 111 lattice planes are also ordered. This gives an additional modulation to the intensity of the ordering reflections. Several laminae are also ordered. This gives an additional modulation of the ordering reflections. Several samples of Ga$_{0.5}$In$_{0.5}$P and Al$_{0.5}$In$_{0.5}$As have been studied experimentally employing synchrotron x-ray radiation. By comparing the experimental and calculated x-ray reciprocal area maps, the structural parameters of the films, such as the mean domain size, the mean thickness of the laminae of both variants, and the average order parameter have been determined.

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