MASS LOADING COMPENSATION FOR FREQUENCY RESPONSE
FUNCTION MEASUREMENTS BY INVERSE RCSA

Kadir Kiran\textsuperscript{a,b}, Harsha Satyanarayana\textsuperscript{b}, Chang Kyu Song\textsuperscript{b,c}, and Tony L. Schmitzb
\textsuperscript{a}Department of Mechanical Engineering, Suleyman Demirel University, Turkey
\textsuperscript{b}Department of Mechanical Engineering and Engineering Science, University of North Carolina at Charlotte, Charlotte, NC, USA
\textsuperscript{c}Department of Ultra Precision Machines and Systems, Korea Institute of Machinery and Materials, Korea

INTRODUCTION
Machining operations can be stable or unstable depending on the cutting parameters and machine-spindle-holder-tool assembly frequency response function (FRF). In some cases, the workpiece FRF can influence the machining behavior as well. Stable, or chatter-free, cutting conditions can be predicted using process stability models [1]. These models require the machine-spindle-holder-tool assembly FRF (and sometimes the workpiece FRF). System FRFs can be obtained by measurement (i.e., modal testing) or models (analytical and/or numerical methods). Schmitz and Donaldson [2] first presented the Receptance Coupling Substructure Analysis (RCSA) method to predict machine-spindle-holder-tool assembly FRFs by coupling individual component FRFs (or receptances). This method reduces measurement time because assembly FRFs can be predicted rather than measured. Subsequent publications have improved the technique [3-12].

In this paper a novel application of RCSA is presented in order to improve FRF measurement accuracy. In modal testing, FRFs are measured using an instrumented hammer to excite the system and (typically) an accelerometer to record the response. However, the measured FRF differs slightly from the actual FRF due to the accelerometer and cable mass. In order to compensate for this mass loading, the inverse RCSA approach is applied here.

RCSA BACKGROUND
RCSA is used to predict an assembly’s receptances by coupling receptances from the individual components. The connections between components can be rigid or flexible with or without energy dissipation (damping) [13]. An example for rigid coupling of two components is displayed in Fig. 1.

For this example, the component direct receptances can be described as $h_{a1a} = \frac{x_{fa}}{f_{fa}}$ (component I) and $h_{b1b} = \frac{x_{fb}}{f_{fb}}$ (component II). The compatibility condition for the rigid coupling is $x_{fb} - x_{fa} = 0$. The equilibrium condition, $f_{fa} + f_{fb} = F_1$, relates the internal (component) forces to the external (assembly) forces. The assembly (III) direct receptance, $H_{11}$, at assembly coordinate, $X_1$, can be expressed as shown in Eq. 1.

$$H_{11} = \frac{X_1}{F_1} = h_{a1a} - h_{a1a} (h_{a1a} + h_{b1b})^{-1} h_{a1a}$$  \hfill (1)

Figure 1. Two component RCSA model: I and II are individual components and III is the assembly.

INVERSE RCSA APPROACH FOR MASS LOADING COMPENSATION
A tool point FRF is measured by modal testing as shown in Fig. 2a. The experimental FRF differs from the actual FRF due to the accelerometer and cable mass. A reduction in the natural frequency(s) and FRF magnitude may be observed, depending on the amount of mass loading.

The accelerometer and cable mass can be compensated using inverse RCSA, where the corresponding RCSA model is depicted in Fig. 2b. In this model, it is assumed that accelerometer is rigidly coupled to the tool point. The measurement provides the assembly receptance, $H_{11} = X_1/F_1$. The accelerometer/cable receptance is $h_{a1a} = \frac{x_{fa}}{f_{fa}}$, while the unknown tool point receptance is $h_{b1b} = \frac{x_{fb}}{f_{fb}}$. The tool point receptance can be determined by rearranging Eq.1 as shown in Eq. 2. This approach is
referred to as inverse RCSA since it is a decoupling, rather than a coupling, step.

\[ h_{tb1b} = -h_{t1a} + h_{t1a} (h_{t1a} - H_{11})^{-1} h_{t1a1} \]  

For this study, the accelerometer/cable is defined as a point mass. The corresponding receptance is provided in Eq. 3, where \( m \) is the mass and \( \omega \) is the frequency (rad/s).

\[ h_{t1a1a} = -1/(m\omega^2) \]  

**RESULTS**

Experiments were performed on two setups: 12.7 mm diameter and 6.35 mm diameter steel rods clamped at one end in a cantilever configuration. Figure 3 displays a photograph of the 12.7 mm diameter rod measurement platform. A split clamp was used to hold the tool at the desired cantilever length. The 6.35 mm diameter rod was clamped in an ER16 collet holder with a CAT-40 interface, which was then secured using a manual draw bolt in a spindle nose attached to a large steel block; see Fig. 4.
For both rod diameters, multiple stickout lengths were selected and measurements were performed with: a medium-size accelerometer (PCB 352A21), a small-size accelerometer (PCB 352C23), and a laser vibrometer (Polytec OFV-534). To obtain the accelerometer/cable masses, measurements were performed using an Ohaus AV264C Adventurer ProAnalytical Balance (0.1 mg resolution). A measurement example is displayed in Fig. 5, where the accelerometer, approximate catenary length of the cable from the tool point measurement (for the cable mass), and the modal wax were included.

A summary of 12.7 mm diameter rod measurement results for both accelerometers at four different stickout lengths is provided in Table 1.

Table 1. 12.7 mm diameter rod results.

<table>
<thead>
<tr>
<th>Stickout (mm)</th>
<th>Vibro. freq. (Hz)</th>
<th>Accel. freq. (Hz)</th>
<th>% error</th>
<th>Comp. freq. (Hz)</th>
<th>% error</th>
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</thead>
<tbody>
<tr>
<td>Medium accelometer (706 mg)</td>
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<tr>
<td>102</td>
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*Stickout lengths are approximate. Different setups were used between the small and medium accelerometers.

Figure 6 shows the medium accelerometer attachment for a stickout length of 102 mm for the 12.7 mm diameter rod. The measurement and compensation results are displayed in Fig. 7, where the accelerometer/cable mass was 706 mg. It is seen that the accelerometer FRF has a lower natural frequency than the vibrometer (non-contact) FRF. Using Eq. 2 and the measured mass, the mass loaded FRF is compensated to remove the mass loading effect. This result matches closely with the vibrometer result (phase error in the vibrometer measurement due to a time delay between the hammer and vibrometer electronics was compensated using the technique described in [14]).

A summary of 12.7 mm diameter rod measurement results for both accelerometers at four different stickout lengths is provided in Table 1.

![Figure 6: Medium accelerometer attached to the 12.7 mm diameter rod using modal wax.](image6)

![Figure 7: Results for 12.7 mm diameter rod with a stickout length of 102 mm using the medium accelerometer.](image7)

![Figure 8: Small accelerometer attached to the 6.35 mm diameter rod using modal wax.](image8)
Figure 8 shows the small accelerometer attached to the 6.35 mm diameter rod. Measurement results are presented in Table 2.

<table>
<thead>
<tr>
<th>Stick-out (mm)</th>
<th>Vibro. freq. (Hz)</th>
<th>Accel. freq. (Hz)</th>
<th>% error</th>
<th>Comp. freq. (Hz)</th>
<th>% error</th>
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<td>-0.12</td>
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*Stickout lengths are approximate. Different setups were used between the small and medium accelerometers.

CONCLUSIONS
This paper described the application of inverse Receptance Coupling Substructure Analysis (RCSA) to mass compensation for accelerometer-based impact testing. The measurement (assembly) FRF was used together with a point mass model for the accelerometer/cable to determine the compensated tool point FRF. Experiments were completed for two tool diameters and multiple stickout lengths for a cantilever configuration. The average percent error in fundamental natural frequency after compensation was -0.03%.

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REFERENCES