INTRODUCTION
The analytical stability lobe diagram offers a predictive capability for selecting stable chip width-spindle speed combinations in machining operations [1-4]. However, the increase in allowable chip width provided at spindle speeds near integer fractions of the system's dominant natural frequency is diminished substantially at low spindle speeds where the stability lobes are closely spaced. Fortunately, the process damping effect can serve to increase the chatter-free chip widths at these low speeds. This increased stability at low spindle speeds is particularly important for hard-to-machine materials that cannot take advantage of the higher speed stability zones due to prohibitive tool wear at high cutting speeds.

This paper presents an analytical solution for milling stability that includes process damping effects. A velocity-dependent process damping force model, which relies on a single coefficient, is applied. The process damping coefficient is identified experimentally for a selected tool-workpiece pair and the effects of tool wear and relief angle are evaluated.

PROCESS DAMPING DESCRIPTION
Interference between the cutting tool’s flank face and the undulations left behind on the cut surface serves as an energy dissipation mechanism and increases stability at low cutting speeds. This energy dissipation at the tool/workpiece interface is recognized as the main source of process damping. The process damping force, $F_d$, in the surface normal ($n$) direction can be expressed as a function of cutter velocity, $\dot{n}$, chip width, $b$, cutting speed, $V$, and a process damping coefficient, $C$ [5]. See Eq. 1.

$$F_d = -C \frac{b}{V} \dot{n}$$

This process damping force, which depends on both the spindle speed-dependent limiting chip width and the cutting speed, is not included in traditional regenerative chatter analytical solutions. Therefore, an iterative solution that includes process damping effects was developed. The empirical process damping coefficient is the only value required to describe the process damping model. This is analogous to the specific cutting force, $K_s$, approach used to model cutting force.

STABILITY ALGORITHM
The process damping force model defined in Eq.1 is applied for an up milling example. The geometry is shown in Fig. 1, where $n$ is the surface normal direction. The projection of the process damping force from the $n$ direction onto the $x$ direction is:

$$F_x = F_d \cos(90 - \phi_{ave}) = -\left( C \frac{b}{V} \cos(90 - \phi_{ave}) \right) \dot{n}.$$ (2)

Substituting $\dot{n} = \cos(90 - \phi_{ave}) \dot{x}$ in Eq. 2 gives:

$$F_x = -\left( C \frac{b}{V} \cos^2(90 - \phi_{ave}) \right) \dot{x}.$$ (3)

The new damping in the converging stability calculation for the $x$ direction frequency response function, $G_x$, is therefore:
\[ c_{\text{new},x} = c_x + C \frac{b}{V} \cos^2(90 - \phi_{\text{ave}}). \]  \( \text{(4)} \)

Note that \( c_x \) is the original damping from the system structural dynamics. The new \( y \) direction damping is:

\[ c_{\text{new},y} = c_y + C \frac{b}{V} \cos^2(180 - \phi_{\text{ave}}). \]  \( \text{(5)} \)

Replacing the original structural damping with \( c_{\text{new}} \) in the system’s frequency response function enables process damping to be incorporated in the traditional regenerative chatter stability analysis.

Tlusty’s time invariant stability model for milling [3] is used to develop analytical stability lobes for the up milling example. Because the new damping value is a function of both the spindle speed-dependent limiting chip width and the cutting speed, the \( b \) and \( \Omega \) vectors must be known in order to implement the new damping value. This leads to the converging nature of the stability analysis that incorporates process damping. The following steps are completed for each lobe in the stability lobe diagram:

1. the analytical stability boundary is calculated with no process damping to identify initial \( b \) and \( \Omega \) vectors
2. these vectors are used to determine the corresponding \( c_{\text{new}} \) vector
3. the stability analysis is repeated with the new damping value to determine updated \( b \) and \( \Omega \) vectors
4. the process is repeated until the stability boundary converges.

To demonstrate the approach, consider the model in Fig. 1 with \( k = 9 \times 10^6 \text{ N/m} \) (stiffness), \( f_n = 900 \text{ Hz} \) (natural frequency), \( \zeta = 0.03 \) (damping ratio), \( K_s = 2000 \times 10^6 \text{ N/m}^2 \) (specific cutting force), \( \beta = 70 \text{ deg} \) (force angle), and \( d = 19 \text{ mm} \) (tool diameter). The stability boundary with no process damping (\( C = 0 \)) is shown in Fig. 2 for 60 lobes. It is observed that the limiting chip width approaches the asymptotic stability limit of 0.50 mm for spindle speeds below 1000 rpm.

Results for the converging procedure with process damping are provided in Fig. 3. Converging behavior is observed for the 10 iterations as the lobes move up and slightly to the right. Although a convergence criterion, such as a threshold percent difference between subsequent minimum values, could be implemented, a practical selection of 20 iterations was applied for the diagrams in this study to ensure convergence. Figure 4 displays the new stability diagram for 60 lobes with \( C = 3.0 \times 10^5 \text{ N/m} \).

**FIGURE 2.** Stability diagram for symmetric up milling model from Fig. 1 with \( k = 9 \times 10^6 \text{ N/m}, f_n = 900 \text{ Hz}, \zeta = 0.03, K_s = 2000 \times 10^6 \text{ N/m}^2, \beta = 70 \text{ deg}, d = 19 \text{ mm} \) and \( C = 0 \).

**FIGURE 3.** Convergence demonstration (10 iterations) for symmetric up milling model from Fig. 1 with \( k = 9 \times 10^6 \text{ N/m}, f_n = 900 \text{ Hz}, \zeta = 0.03, K_s = 2000 \times 10^6 \text{ N/m}^2, \beta = 70 \text{ deg}, d = 19 \text{ mm} \) and \( C = 3.0 \times 10^5 \text{ N/m} \).
In order to provide convenient control of the system dynamics, a single degree-of-freedom, parallelogram leaf-type flexure was constructed to provide a flexible foundation for individual AISI 1018 steel workpieces; see Fig 5. Because the flexure compliance was much higher than the tool-holder-spindle-machine, the stability analysis was completed using only the flexure’s dynamic properties. A radial immersion of 50% and a feed per tooth of 0.05 mm/tooth was used for all conventional (up) milling tests.

The effect of the cutting insert relief angle was examined using 15 and 11 deg relief angle inserts. Wear effects were evaluated by milling with new and moderately worn inserts. The flexure dynamics were also adjusted to determine the sensitivity of the process damping coefficient to changes in the system dynamics.

**Process Damping Coefficient Identification**

A grid of test points at low spindle speeds was selected to investigate the process damping behavior. Based on the stable/unstable cutting test results, a single variable residual sum of squares (RSS) estimation was applied to identify the process damping coefficient that best represented the experimental limiting axial depth of cut, $b_{lim}$. The results of the coefficient identification method are depicted in Figure 6.

**RESULTS**

**Cutting Tests**

For a 50% radial immersion up milling cut, the stability boundary was discovered to increase as the spindle speed decreased and a value of for the process damping coefficient was obtained. Replacing the original 15 deg relief angle insert with an 11 deg relief angle insert and repeating the cutting tests revealed that the process damping coefficient increased by approximately
30% under low wear conditions. This indicates that the decreased relief face angle increases process damping. The results for the low wear tests are summarized in Table 1.

**TABLE 1.** Comparison of process damping coefficients for low wear tests.

<table>
<thead>
<tr>
<th>Relief angle (deg)</th>
<th>C (N/m) for the 228 Hz setup</th>
<th>C (N/m) for the 156 Hz setup</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2.5×10⁵</td>
<td>2.6×10⁵</td>
</tr>
<tr>
<td>11</td>
<td>3.3×10⁵</td>
<td>3.3×10⁵</td>
</tr>
</tbody>
</table>

Tests were also completed using moderately worn inserts. Under worn conditions, the process damping coefficient increased by approximately 15% compared to new conditions. The process damping values obtained using moderately worn inserts are presented in Table 2. Finally, the dynamic properties of the system were adjusted by increasing the mass fixed to the flexure. No appreciable change in the process damping coefficient was observed.

**TABLE 2.** Comparison of process damping coefficients for moderate wear tests.

<table>
<thead>
<tr>
<th>Relief angle (deg)</th>
<th>C (N/m) for the 228 Hz setup</th>
<th>C (N/m) for the 156 Hz setup</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
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<td>3.4×10⁵</td>
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<tr>
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<td>3.8×10⁵</td>
</tr>
</tbody>
</table>

**Repeatability**

Repeat testing was performed using the 19.05 mm diameter, 11 deg relief angle cutting tool in order to observe the variability in the process damping coefficient. A series of three additional cutting tests were performed on the 228 Hz system with an unworn insert. Assuming a normal distribution, a two-sided 90% confidence level was computed for this small sample size. The confidence interval for the population mean was: \( C = (3.2\pm 0.15) \times 10^5 \) N/m. Figure 7 illustrates the corresponding confidence region.

**CONCLUSIONS**

An analytical solution for machining stability while considering process damping was developed, where the process damping model was dependent on a single empirical coefficient, C. Stability testing was completed using a single degree-of freedom flexure to identify the process damping coefficient for low-speed milling of AISI 1018 steel under various conditions. It was demonstrated that a reduction in the relief angle and an increase in flank wear of the cutting edge increased the process damping coefficient.

Process damping is particularly important for hard-to-machine materials, such as titanium, nickel super alloys, and hardened steels. In these instances, tool wear generally prohibits higher surface speeds and the use of the large stable zones available at high spindle speeds. This limits the spindle speed to low values, which decreases the material removal rate. However, by exploiting process damping, higher stable axial depths and material removal rates can be achieved.

**REFERENCES**


