The Design of a Multiple Degree of Freedom Flexure Stage with Tunable Dynamics for Milling Experimentation

Mark A. Rubeo*, Kadir Kiran*, and Tony L. Schmitz†
The University of North Carolina at Charlotte, Charlotte, NC
mrubeo@uncc.edu, kkiran@uncc.edu, tony.schmitz@uncc.edu

Abstract
This paper describes the design of an apparatus with tunable dynamics for use in milling experimentation. The design includes a series of nested, orthogonal flexure stages which provide compliance in two degrees of freedom. By modifying a number of the structural components, the effective mass, stiffness, and damping of the system can be varied to suit experimental needs. It is shown that the variability allowed by the design can enable a wider range of milling experimentation by more closely modeling the response of typical machine-tool-workpiece systems.

Keywords: Milling, dynamics, chatter, flexure, eddy current damping

1 Introduction

Modeling of the milling process is motivated by the requirement to achieve the specified dimensional accuracy and surface quality of workpieces. Milling simulation models allow the user to select key machining parameters, such as spindle speed and depth of cut, for optimized performance pre-process (Schmitz & Smith, 2008). To implement and experimentally validate these simulations, the dynamic response of the system must be known. The system dynamics are dominated by the most flexible component of the machine-tool-workpiece system.

Typical validation platforms for milling experiments consist of single degree of freedom (DOF) flexure stages on which a workpiece is mounted; see Figure 1. Because the flexure stage is significantly more flexible than the cutting tool, its dynamics dominate the overall system response, and the tool may be considered as a rigid body. As a result, milling experiments are facilitated because the overall system response has been simplified.

However, as milling simulation models increase in complexity there is an increasing need for validation platforms that more closely mimic machine-tool-workpiece systems that are common in...
industry. The typical validation platform (i.e., flexure stage) may be augmented to include additional degrees of freedom and variable dynamics (i.e., mass, stiffness, and damping). In this paper, the design of a two degree of freedom flexure stage with variable stiffness, natural frequency, and viscous damping is described.

![Figure 1. Parallelogram, leaf-type flexure used for milling experiments.](image)

2 Design Concept

The validation platform is composed of two parallelogram, leaf type flexures in an orthogonal, nested orientation; see Figure 2. Experimental workpieces are mounted to the upper platen, which has flexibility in the x and y directions, via a grid of M5 threaded holes. By selecting the leaf flexure’s geometry (i.e., length, width, and thickness) and the mass of the upper and lower platen, the stiffnesses and natural frequencies of the validation platform may be tuned to meet experimental requirements. In the configuration shown in Figure 2, the geometry of the flexures in the x and y directions are identical. However, different flexure geometries for each direction may be selected such that the dynamic response of the validation platform varies by direction. This design feature allows for milling experiments to be conducted on a two DOF system where one direction is significantly more flexible than the other.

In addition to tunable stiffness and natural frequency, the validation platform incorporates a pair of eddy current dampers to provide viscous damping. The damping force provided by the eddy current dampers can be described analytically; see Section 3.2. It is shown that the damping force is a function of magnetic field strength. By varying the gap between the opposing faces of the magnet yoke assembly, the magnetic field strength, and similarly, the viscous damping forces may be varied. The damping force acts on a copper conductor which is rigidly fixed to the upper platen; see Figure 3. By virtue of the orientation of the magnetic field and motion of the upper platen, the viscous damping force acts in both the x and y directions.

The upper platen, lower platen, and frame are composed of 6061-T6 aluminum, and the base plate is composed of 1018 steel which has been ground to provide a flat mounting surface for the flexure stage. Other materials include zinc-coated, alloy steel threaded fasteners and super-conductive 101 copper for the eddy current damper.
Figure 2. Multi-DOF flexure stage with eddy current dampers.
The yoke assembly, shown in Figure 4, houses a pair of opposing, three by three arrays of Neodymium Iron Boron (NdFeB) magnets. By selecting a ferromagnetic material (i.e., mild steel) for the yoke assembly, a magnetic circuit is formed and the magnetic flux lines are directed in the gap between the magnet arrays. In this manner, the strength of the magnetic field is enhanced. Additionally, the magnetic field strength is adjustable by varying the width of the gap between the magnet arrays. The mechanism for adjusting the gap width, shown in Figure 4, is conceptually similar to the vise stop which is commonly found in machine shops for repeatable workpiece positioning. The upper plate of the yoke assembly, which is rigidly fixed to the supporting structure of the flexure stage, houses two 8 mm dowel pins which are fixed in place by an interference fit. These dowel pins mate with the lower plate of the yoke assembly via a sliding fit, and a series of Belleville washers provides a spring force which puts tension on the adjustment screw. The adjustment screw is accessible while the flexure stage is fully assembled facilitating on-the-fly variation of the viscous damping force.
Figure 4. Steel yoke assembly including exploded view of mechanism for adjusting the magnetic field strength (i.e., magnet gap width).
3 Design Analysis

3.1 Flexure Stiffness and Natural Frequency

A thorough treatment of flexure design analysis is given by (Smith, 2000). For brevity a complete derivation of the relevant design equations shall be omitted from this paper. However, the stiffness of a parallelogram, leaf type flexure, \( k \), where the force is applied at one-half the length of the leaf flexure is given by:

\[
k = 2Er^3
\]

where \( E \) is Young’s modulus of the flexure material and \( r, t, \) and \( l \) are the flexure width, thickness, and length, respectively. It is noteworthy that the flexure stiffness is a cubic function of the thickness to length ratio. The proposed flexure stage has been designed to have a variable stiffness depending upon the leaf geometry. Two embodiments of the flexure stage stiffness, and corresponding leaf flexure geometry, are given in Table 1 as an example. The given examples consider a total of 4 leaf flexures composed of hardened spring steel. In this case, the stiffness is tuned by varying the length of the flexure, but in principle the stiffness may be altered by varying any of the parameters given in Equation (1).

<table>
<thead>
<tr>
<th>Flexure Geometry</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
<th>Young’s Modulus (GPa)</th>
<th>Mechanism Stiffness (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45</td>
<td>35</td>
<td>3.175</td>
<td>200</td>
<td>9.83 x 10^6</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>35</td>
<td>3.175</td>
<td>200</td>
<td>1.04 x 10^6</td>
</tr>
</tbody>
</table>

The equivalent mass, \( m_{eq} \), and undamped natural frequency, \( \omega_n \), of the flexure stage are given as:

\[
m_{eq} = m_p + \frac{26}{35} m_t
\]

\[
\omega_n = \sqrt{\frac{k}{m_{eq}}}
\]

where \( m_p \) is the total mass of the moving platen, leaf clamps, copper conductors, and fasteners and \( m_t \) is the mass of the leaf flexures. Because the two, orthogonal DOFs are in a stacked (i.e., nested) arrangement, the equivalent masses (i.e., moving masses) are not the same. When the x direction, defined in Figure 2, is set in motion, there is a larger moving mass. Therefore for a fixed value of stiffness, the natural frequency of the x and y direction vibration will be different. For the high stiffness arrangement, given in Table 1, the equivalent masses and natural frequency for the two DOFs have been calculated and the results are summarized in Table 2. It is noteworthy that the x direction, which has a
higher equivalent mass, has a lower natural frequency. If equivalent natural frequencies in the x and y 
directions are desired, the mass and stiffnesses can be tuned accordingly.

**Table 2.** Equivalent mass and undamped natural frequency in the x and y directions for the high stiffness 
arrangement.

<table>
<thead>
<tr>
<th></th>
<th>( m_p )</th>
<th>( m_t )</th>
<th>( m_{eq} )</th>
<th>( k )</th>
<th>( f_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X - direction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_p )</td>
<td>2.92 kg</td>
<td>0.23 kg</td>
<td>3.09 kg</td>
<td>( 9.83 \times 10^6 ) N/m</td>
<td>284 Hz</td>
</tr>
<tr>
<td>( m_{eq} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Y - direction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_p )</td>
<td>2.28 kg</td>
<td>0.23 kg</td>
<td>2.45 kg</td>
<td>( 9.83 \times 10^6 ) N/m</td>
<td>319 Hz</td>
</tr>
</tbody>
</table>

### 3.2 Eddy Current Damping

The viscous damping force, which is velocity dependent, is generated by an eddy current damper 
and can be calculated analytically. Figure 5 illustrates the motion of a nonmagnetic conductor, which is 
fixed to the upper platen, relative to the magnet arrays housed within the steel yokes. Because the 
relative motion is orthogonal to the magnetic flux direction, eddy currents are generated within the 
conductor. The eddy current density, \( \vec{J} \), depends upon the conductivity, \( \sigma \), and the cross product of the 
velocity, \( \vec{v} \), and magnetic field, \( \vec{B} \) according to Equation (4). The viscous damping force is then 
calculated as the volume integral of the cross product of the eddy current density and magnetic field; 
see Equation (5). The resulting viscous damping force acts in the direction opposing the velocity.

\[
\vec{J} = \sigma (\vec{v} \times \vec{B}) \quad (4)
\]

\[
\vec{F} = \int_v (\vec{J} \times \vec{B}) \, dV \quad (5)
\]

The magnitude of the viscous damping force is given by:

\[
F = [\sigma \delta B^2 S(\alpha_1 + \alpha_2)] v = cv \quad (6)
\]

where \( \delta \) is the thickness of the conductor, \( B \) is the magnetic field strength, \( S \) is the area of the magnet 
array, \( \alpha_1 \) incorporates surface charge effects, \( \alpha_2 \) incorporates end charge effects from the finite width 
conductor, and \( v \) is the velocity magnitude (Bae, et al., 2005). This viscous damping coefficient, \( c \), 
enables model-based damping prediction and selection for milling operations.
The design parameters for a single eddy current damper (i.e., the proposed design has two) are listed in Table 3. Using a nominal magnet array gap of 12 mm, the magnetic field strength was estimated using finite element analysis. It is important to note that the magnetic field strength is not constant throughout the entire gap between the magnet arrays. Therefore, the viscous damping coefficient depends upon the average magnetic field strength within the copper conductor. The estimated magnetic field strength is taken from the center of the magnet array and halfway between the opposing faces. For the parameters listed in Table 3 the calculated value of the viscous damping coefficient is 201 \( (N \cdot s)/m \). Given that there are a pair of eddy current dampers, arranged symmetrically about the upper platen, the total damping coefficient for a nominal magnet array gap of 12 mm is 402 \( (N \cdot s)/m \).

Table 3. Eddy current damper design parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>(5.96 \times 10^7 \text{ A/V-m})</td>
</tr>
<tr>
<td>( \delta )</td>
<td>10 mm</td>
</tr>
<tr>
<td>( B )</td>
<td>5700 Gauss (0.57 T)</td>
</tr>
<tr>
<td>( S )</td>
<td>(5.8 \times 10^{-3} \text{ m}^2)</td>
</tr>
<tr>
<td>( \alpha_1^* )</td>
<td>0.500</td>
</tr>
<tr>
<td>( \alpha_2^* )</td>
<td>-0.321</td>
</tr>
</tbody>
</table>
The corresponding dimensionless damping ratio, $\zeta$, in the x and y directions, calculated according to Equation (7), are 0.0365 (3.65%) and 0.041 (4.1%), respectively. Because the dimensionless damping ratio depends upon both stiffness and effective mass (both of which are adjustable) the dimensionless damping ratio can be further tuned as needed for milling experiments.

$$\zeta = \frac{c}{2\sqrt{k m_{eq}}} \tag{7}$$

As previously discussed, the eddy current damper is designed with an adjustable gap width, and, as a result, the viscous damping coefficient is variable. For the high stiffness flexure stage embodiment (i.e., $k = 9.83 \times 10^6 \text{ N/m}$), Figure 6 illustrates the change in viscous damping (i.e. dimensionless damping ratio) with magnet array gap width. The dimensionless damping ratio (and magnetic field strength) decreases linearly as the gap width of the magnet array increases. For the proposed design, the gap width can be varied from approximately 11.5 mm to 19.5 mm.

![Figure 6. Dimensionless damping ratio as a function magnet array gap for the high stiffness embodiment.](image)

### 3.3 System Dynamics

The vibration response of the flexure stage to a periodic input force, such as the interrupted cutting which occurs during milling, can be characterized by its complex valued frequency response function (FRF). Equation (8) gives the FRF for a single degree of freedom system. The FRF highlights two key features of the forced vibration: (1) that the force input and subsequent vibration share the same
frequency and (2) the amplitude of vibration relative to the force input. Typically, the FRF is represented as a magnitude and phase or real and imaginary components.

\[
\frac{X}{F} = \frac{1}{-m\omega^2 + i\omega + k}
\]  

(8)

The magnitude of the flexure stage FRF for the x and y directions are shown in Figure 7. The amplitude of the vibration response reaches a maximum at the natural frequencies, also commonly referred to as resonance, which were calculated as 284 Hz and 319 Hz for the x and y directions, respectively. To illustrate the effect of the eddy current dampers, the FRF for several values of damping are shown. It is noteworthy that in the case where there is no eddy current damping, the flexure stage vibration amplitude at resonance is significantly higher than when the eddy current damper is attached. This behavior is characteristic of flexure systems because the inherent solid damping is low.

![Figure 7](image_url)

**Figure 7.** Magnitude of the flexure stage FRF in the x and y directions for the high stiffness embodiment with different damping values.

Figure 8 illustrates the effect of increased viscous damping, by varying the magnet array gap width, on the frequency response of the flexure stage. It is notable that the vibration amplitude per unit force decreases as viscous damping increases.
Figure 8. Magnitude of the flexure stage FRF showing the effect of increased viscous damping.

4 Experimental Evaluation of System Dynamics

Modal testing, also commonly referred to as impact testing, will be utilized to identify the actual stiffness and damping of the flexure stage. An instrumented hammer will be used to excite the structure over a broad frequency range, and the response will be measured using a low-mass accelerometer. Modal parameters (i.e., mass, stiffness, and damping ratio) will be extracted from the single degree of freedom frequency response function using the peak picking technique. For brevity the technique is not described here, but a detailed description is given in (Schmitz & Smith, 2008). An example of a modal testing setup for another flexure stage is shown in Figure 9.
5 Discussion

This paper describes the design, analysis, and experimental evaluation of a two DOF validation platform with tunable dynamics for milling experimentation. The nested, parallelogram leaf type flexure stage and eddy current damper mechanical design were discussed in detail including materials selected for all components. Further, a detailed design analysis was included with all relevant design equations and reference materials. Finally, the method of experimentally validating the design was presented.

6 Contributions

Mark Rubeo (50%)
- Final Mechanical Design
- Analysis
- Manufacture

Kadir Kiran (50%)
- Preliminary mechanical design
- Manufacture
- Testing (to be completed)

References

