Permeability and Permittivity Extraction Issues For Non-Foster and Active Metamaterials

Joshua W. Shehan, John M. C. Covington III, Varun S. Kshatri, Thomas P. Weldon, and Ryan S. Adams
Department of Electrical and Computer Engineering
University of North Carolina at Charlotte
Charlotte, NC, USA
tpweldon@uncc.edu

Abstract—**Although a variety of methods have been proposed for the extraction of effective permittivity and permeability of metamaterials, certain underlying passivity constraints and assumptions would not be suitable for non-Foster metamaterials that incorporate active devices and power sources. Moreover, recent arguments suggest that a common passivity constraint in extraction methods does not resolve solution branch ambiguities. To address these issues, the fundamental principles of parameter extraction are revisited for the case of active materials where passivity cannot be assumed. The analysis follows along the lines of Nicolson-Ross-Weir approaches, where parameters are extracted from measured two-port S-parameters. It is shown that a convergence constraint for active materials requires that the magnitude of the product of the transmission coefficient and reflection coefficient must be less than unity. This allows metamaterials with gain, and simulation results are provided for a slab of active material that exhibits gain and satisfies the constraint.**

I. INTRODUCTION

Recent experimental and theoretical results suggest that non-Foster circuits offer the potential to create practical wideband metamaterials and devices [1], [2]. To characterize such systems, extraction algorithms are used to compute parameters such as effective permittivity and effective permeability. However, many of these extraction methods employ passivity constraints that are not generally applicable to active metamaterials and devices that incorporate power sources and non-Foster circuits. Such passivity constraints are commonly used in various stages of these algorithms to select roots, or wavenumber sign, or intrinsic impedance sign, or branches of solutions [3]–[6]. Such assumptions are not suitable for active materials and devices, such as those containing non-Foster circuits. Moreover, recent arguments question whether passivity constraints resolve branch ambiguities in earlier methods and point out subtle differences between notions of *equivalent* permittivity and *effective* permittivity [7].

To address these issues, fundamental principles are revisited for the case of active materials where passivity cannot be assumed. Since many parameter extraction methods follow a Nicolson-Ross-Weir approach, the analysis proceeds along similar lines [4], [8], [9]. In this approach, a slab of possibly active material is considered, where parameters are extracted from measured two-port S-parameters. The analysis leads to a convergence constraint that is similar to the aforementioned passivity constraint, where the product of the magnitude of transmission and reflection coefficient must be less than unity, a constraint similar to the cavity gain requirements of a laser.

In the next section, the proposed extraction constraint is derived for a planar slab of active material in free space. The subsequent section presents simulation results and extraction results for an active metamaterial slab that exhibits gain.

II. ANALYSIS AND PROPOSED CONSTRAINTS

To consider the case of parameter extraction for active materials, it is useful to revisit first principles for a planar slab of material in free space, as illustrated in Fig. 1. An incident wave E_i in vacuum impinges on the surface of the material from the left side. At the left-hand interface between vacuum and the material, a first reflection ΓE_i occurs, and transmitted wave $(1+\Gamma)E_i$ travels into the material, where the time factor $e^{j\omega t}$ is suppressed. The reflection coefficient is $\Gamma = (\eta - \eta_\circ)\,/\,(\eta + \eta_\circ)$, η is the impedance of the material, and $\eta_\circ = 377\,\Omega$. After traveling through the material to the right-hand interface, the wave $(1+\Gamma)T\,E_i$ impinges on the right-hand interface, where $T = e^{-jk(\omega)d}$ is the one-way transmission constant of the material with thickness d and frequency-dependent wavenumber $k(\omega)$. At the right-hand interface, the reflected wave is $-\Gamma(1+\Gamma)T\,E_i$, and the transmitted wave is $(1-\Gamma^2)T\,E_i$. The reflected wave $\Gamma(1+\Gamma)T^2\,E_i$ arrives at the left-hand interface. The component $\Gamma(\Gamma^2-1)T^2\,E_i$ is transmitted to the left-hand

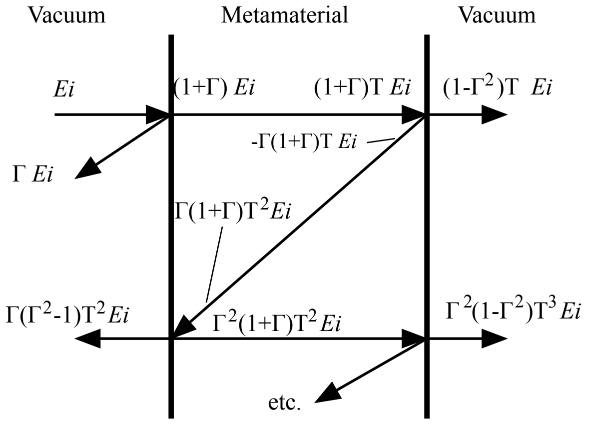

Fig. 1. Diagram of multiple reflections within a slab of material suspended in vacuum.

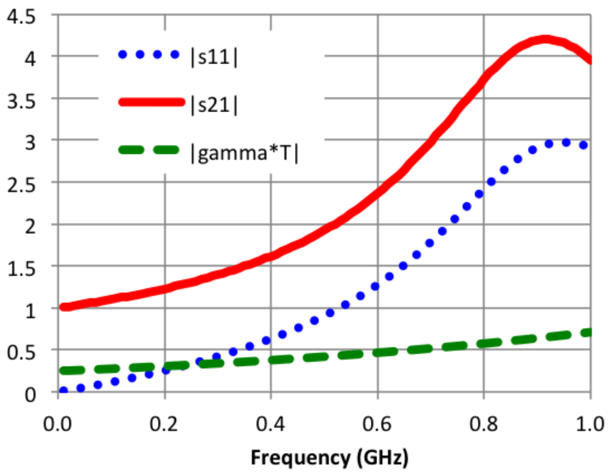

Fig. 2. Simulated $|S_{11}|$ (dotted blue), $|S_{21}|$ (solid red), and constraint parameter $|\Gamma\, T|$ (dashed green) for a slab $d = 0.03$ m thick with $\epsilon = (2 + j3)\epsilon_\circ$, and permeability $\mu = 3.5\mu_\circ$.

vacuum, and reflected wave $\Gamma^2(1+\Gamma)T^2\,E_i$ travels to the right, where the transmitted wave into the right-hand vacuum is $\Gamma^2(1-\Gamma^2)T^3\,E_i$. Summing all the reflections and dividing by the incident wave E_i yields S-parameter S_{11}:

$$\begin{aligned} S_{11} &= \Gamma\left(1+\left(\Gamma^2-1\right)T^2\sum_{n=0}^{\infty}\left(\Gamma^2T^2\right)^n\right)\\ &= \frac{\Gamma\left(1-T^2\right)}{1-\Gamma^2T^2}\;; \qquad \text{if } |\Gamma\, T|<1\,, \end{aligned} \tag{1}$$

where the required condition for convergence of the sum, $|\Gamma\, T| < 1$, is now a strict constraint imposed on extraction algorithms for active media such as metamaterials incorporating non-Foster devices. Note that this constraint is less stringent than a passivity constraint which would require $|\Gamma| < 1$ and $|T| < 1$, and the proposed constraint allows for media that exhibit gain, i.e., $|T| > 1$. Similarly, S_{21} follows:

$$S_{21} = T\left(1-\Gamma^2\right)\sum_{n=0}^{\infty}\left(\Gamma^2T^2\right)^n = \frac{T\left(1-\Gamma^2\right)}{1-\Gamma^2T^2} \tag{2}$$

with convergence condition $|\Gamma\, T| < 1$ as before. Also, note that (1) and (2) are the typical starting points for Nicolson-Ross-Weir based approaches to parameter extraction [3]–[9].

III. RESULTS

To demonstrate extraction in an active material with gain, the configuration of Fig. 1 was simulated from 10 MHz to 1 GHz for a slab of thickness $d = 0.03$ m, permittivity $\epsilon = (2+j3)\epsilon_\circ$, and permeability $\mu = 3.5\mu_\circ$, $\mu_\circ = 1.26\times10^{-6}$ H/m, and $\epsilon_\circ = 8.85\times10^{-12}$ F/m. The simulated S-parameters are shown in Fig. 2, where the blue dotted curve is the magnitude of S_{11}, the solid red curve is the magnitude of S_{21}, and the dashed green curve shows the aforementioned extraction constraint $|\Gamma\, T|$. Since $|S_{11}| > 1$ and $|S_{21}| > 1$ above 500 MHz, the active material exhibits gain for transmitted and reflected waves. In addition, $|\Gamma\, T| < 1$ over the full frequency range, and so (1) and (2) converge over the full band and thus Nicolson-Ross-Weir based extraction should apply.

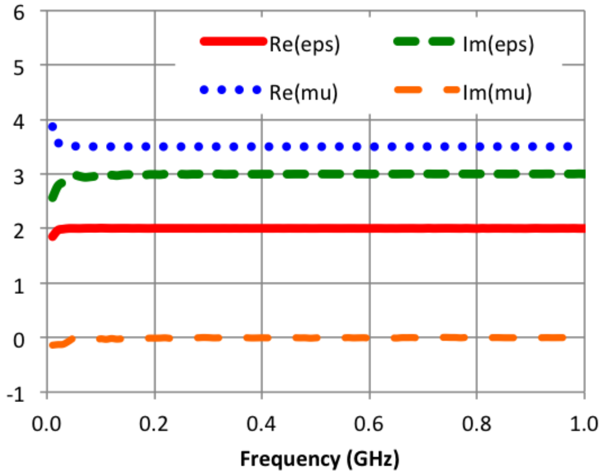

Fig. 3. Extracted parameters closely matching the parameters of the material with $\epsilon = \epsilon_r\epsilon_\circ = (2+j3)\epsilon_\circ$, and permeability $\mu = \mu_r\mu_\circ = 3.5\mu_\circ$.

Since the constraint $|\Gamma\, T| < 1$ was satisfied, parameter extraction was performed as adaptations of two different extraction methods [4], [5], and both methods yielded nearly identical results for this example. Fig. 3 shows that the extracted parameters closely match the material parameters, even though the material is clearly not a passive medium. The solid red curve shows the extracted real part of ϵ_r, the dotted blue curve is the extracted real part of μ_r, the dashed green curve is the extracted imaginary part of ϵ_r, and the orange long-dashed curve is the extracted imaginary part of μ_r.

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