Wideband Negative Permeability Metamaterial with Non-Foster Compensation of Parasitic Capacitance

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Abstract—The analysis and simulation of negative effective permeability of a magnetic metamaterial is presented, including parasitic effects. Beyond known issues of non-Foster circuit stability, such parasitics can limit bandwidth improvement. Based on the analysis, ideal non-Foster elements are added to split rings to achieve broadband negative effective permeability while compensating parasitic effects. Results indicate that both a negative capacitance and negative inductance are needed to achieve negative permeability from 0.5 to 4.5 GHz.

I. INTRODUCTION

Magnetic metamaterial unit cells are commonly narrowband and dispersive. However, the appropriate use of non-Foster elements can increase the bandwidth of metamaterials, in principle [1], [2]. Therefore, the present work addresses the deleterious effects of parasitic fringe capacitance on the bandwidth of a single split-ring resonator when loaded with ideal non-Foster circuit elements. Analysis of the parasitics leads to modified equations for effective permeability, and simulation results confirm the potential for significantly improved bandwidth. In addition, the following discussion focuses only on the bandwidth problem with ideal elements, necessarily deferring more complex issues of stability with practical circuit elements for future investigation. Notwithstanding these limitations, the following analysis and simulation results illustrate the importance of mitigating such parasitics.

II. WIDEBAND UNIT CELL WITH PARASITIC EFFECT

For simplicity, a lossless single split-ring resonator (SSRR) is used to illustrate the influence of parasitic fringe capacitance on the effective permeability of the metamaterial when using non-Foster elements. Consider a SSRR as shown in Fig. 1 centered in a unit cell with dimensions $l_x$, $l_y$, $l_z$. The area of the SSRR is $A_R$ and the incident magnetic field $H_0$ is parallel to the axis of the SSRR. Due to the change in the magnetic field, a voltage $v_g$ appears across the gap of the ring. The gap in the split-ring can be modeled as a capacitance $C_g$. The current $i_r$ in the ring and through capacitance $C_g$ is then

$$i_r = C_g \frac{dv_g}{dt} = -C_g \frac{d^2(\Phi_0 + \Phi_R)}{dt^2} = -\Phi_0 \frac{s^2C_g}{1 + s^2L_RC_g},$$

where $s$ is the Laplace complex angular frequency, $L_R = \Phi_R/i_r$ is self-inductance, $v_g = -d(\Phi_0 + \Phi_R)/dt$, $\Phi_0$ is the incident magnetic flux, and $\Phi_R$ is the magnetic flux due to $i_r$.

The well-known result in (1) describes the conventional narrowband behavior of a SSRR, where the magnetic resonance frequency can be defined as $\omega_m = 1/\sqrt{L_RC_g}$.

Next, consider replacing gap capacitance $C_g$ with a positive inductance $L_g$ with reactance $X_L = j\omega L_g$. The ring current $i_s$ then becomes

$$i_r = \frac{1}{L_g} \int v_g dt = -\frac{1}{L_g} (\Phi_0 + \Phi_R) = -\Phi_0 \frac{1}{L_g + L_R}.$$  (2)

Comparing (1) with (2), the current in the split-ring is now frequency independent and broadband behavior is possible with proper choice of inductance $L_g$.

In practice, however, capacitance $C_g$ cannot be removed completely and some parasitic fringe capacitance $C_{FG}$ will remain. As a result, the equivalent circuit in the gap of the split-ring is now a parallel combination of inductance $L_g$ and fringe capacitance $C_{FG}$. Modifying (2) with $C_{FG}$ yields

$$i_r = i_{C_{FG}} + i_{L_g} = C_{FG} \frac{dv_g}{dt} + \frac{1}{L_g} \int v_g dt,$$  (3)

where $i_{C_{FG}}$ is the current through fringe capacitance $C_{FG}$, and $i_{L_g}$ is the current through inductance $L_g$. Substituting $v_g = -d(\Phi_0 + \Phi_R)/dt$ in (3), taking the Laplace transform, and including self-inductance $L_R$ yields

$$i_r = -\Phi_0 \frac{1 + s^2C_{FG}L_g}{L_R + L_g(1 + s^2C_{FG}L_R)}.$$  (4)

The result in (4) indicates that two resonance frequencies exist.

To find effective permeability, magnetic dipole moment is used. The current in the SSRR creates a magnetic dipole moment $m = i_sA_R$. The magnetization $M$ is then the magnetic

Fig. 1. Single split-ring resonator (SSRR) used as an example of a magnetic unit cell.
and found to strongly limit the bandwidth of negative effective
result when the negative capacitance is removed.

GHz. The blue dashed and green (triangle) curves depict the
(square) curves illustrates wideband behavior from 0.5 to 4.5
resonance occurs near 2.5 GHz. The black dotted and black dashed
describe the conventional narrowband behavior. The magnetic
For both Figs. 3 and 4 the red solid and purple (circle) curves
were only loaded with a negative inductance.

negative capacitance was removed and all three SSRR devices
wideband behavior as predicted in (6). In the final case, the
three SSRR devices were loaded with negative capacitance
without non-Foster circuit elements. In the second case, all
simulated. The first case used conventional SSRR devices
with three SSRR devices in a parallel-plate waveguide with
wideband negative, wideband negative effective permeability possible when
and

\[ M = \frac{i R A R}{l x y z}, \]
Since
\[ M = \chi_m H, \mu_r = 1 + \chi_m, \text{ and } \Phi_0 = \mu_0 H_0 A_R, \]
the relative permeability, \( \mu_r \), equals
\[
\mu_r = 1 - \mu_0 \frac{A_{R}^2}{l x y z} \frac{1}{L_R + \omega^2 C_F g L_g},
\]
where \( \chi_m \) is the magnetic susceptibility, \( \omega \) is the angular
frequency, \( \mu_0 = 1.26 \times 10^{-6} \text{ H/m} \) is the permeability of free
space, and \( s = j \omega \) was used.

Finally, the parasitic fringe capacitance \( C_{Fg} \) can theoretically be canceled by adding a parallel negative capacitance of
equal value such that (5) becomes
\[
\mu_r = 1 - \mu_0 \frac{A_{R}^2}{l x y z} \frac{1}{L_R + L_g},
\]
and \( \mu_r \) once again becomes frequency independent, making
wideband negative effective permeability possible when \( L_g \) is
negative, \( L_R + L_g > 0 \), and \( L_R + L_g \approx 0 \), according to (6).

III. SIMULATIONS

The metamaterial structure shown in Fig. 2 was simulated
with three SSRR devices in a parallel-plate waveguide with
perfect electric conductor top and bottom walls and with
perfect magnetic conductor side walls. Three cases were
simulated. The first case used conventional SSRR devices
without non-Foster circuit elements. In the second case, all
three SSRR devices were loaded with negative capacitance of
-47 fF and negative inductance of -16.7 nH to confirm
wideband behavior as predicted in (6). In the final case, the
negative capacitance was removed and all three SSRR devices
were only loaded with a negative inductance.

For the three cases simulated, \( S_{21} \) is plotted in Fig. 3 and
extracted relative permeability is shown in Fig. 4 (using [3]).
For both Figs. 3 and 4 the red solid and purple (circle) curves
describe the conventional narrowband behavior. The magnetic
resonance occurs near 2.5 GHz. The black dotted and dashed
(square) curves illustrates wideband behavior from 0.5 to 4.5
GHz. The blue dashed and green (triangle) curves depict the
result when the negative capacitance is removed.

IV. CONCLUSION

The deleterious effects of fringe capacitance were analyzed and
found to strongly limit the bandwidth of negative effective
permeability in non-Foster loaded split ring resonators. The
analysis and simulation results show that a non-Foster load
with both negative inductance and negative capacitance is
required for wideband behavior. Related results are in [4].

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