

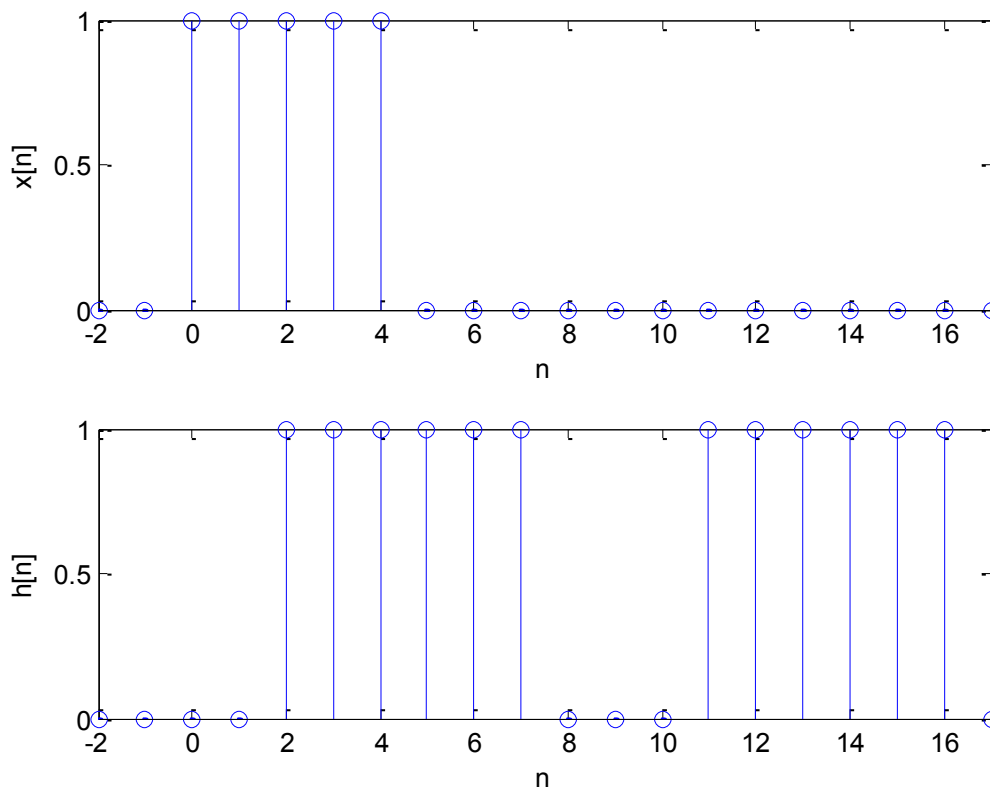
Problem Set 3
Assigned: 2/1/2013 Due: 2/8/2013

Suggested Readings: O+W Sections 2.1 and 2.3 (Pay special attention to Examples 2.3-2.5), O+S Section 2.3 (Pay close attention to Example 2.11).

Problem 1

Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals

- a) $x[n]$ and $h[n]$ as shown below



- b) $x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$ and $h[n] = 4^n u[2-n]$

Problem 2

Determine if the following impulse responses of LTI systems are causal and/or stable. Justify your answers.

- a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$
- b) $h[n] = 0.8^n u[n + 2]$
- c) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$

Problem 3

In Problem Set 2, you found the following difference equation approximation for the LR circuit:

$$\frac{x[n]}{R} = ay[n] - by[n - 1]$$

Where $R = 10\Omega$, $L = 1mH$, and ΔT was up to you. Recall that $a = \left(\frac{L}{R\Delta T} + 1\right)$ and $b = \frac{L}{R\Delta T}$. The circuit is at rest before time $n = 0$.

- a) Find the *impulse response* of this system. To do so, let $x[n] = \delta[n]$. This is different than solving most difference or differential equations since the input to system is zero in steady state, meaning that the particular and homogeneous solutions are the same. In this case, proceed in two steps:
 - a. Find the homogeneous solution.
 - b. Solve the difference equation directly at $n = 0$ to find the initial value $i[0]$.
 - c. Use your results from Steps a and b to find the total impulse response.
- b) Call the solution to part a the impulse response $h[n]$. Now, assume the input is $x[n] = V_{IN}u[n]$. Use the convolution sum to find the response to this input. Compare this result to the solution obtained in Problem Set 2, Problem 3. Explain why the two answers are the same. In other words, provide a real explanation, not simply a statement that “Yes, they are the same, or no, they are different).

Problem 4

In partnership with one of our industrial partners, we’ve been developing a model for the thermal performance of small buildings. One of the key inputs to the building model is the solar irradiance, which is measured in measured in W/m^2 . Often, this data is very “noisy” as the result of quickly moving clouds that pass in front of the sun. In general, the sun’s irradiance can change much, much faster than the temperature of a room. Our desire was thus to determine the response to the “smoothed” irradiance. To smooth out the rapid variations in the measured data, we tried several filters, one of which was the “moving average filter.” If we consider the irradiance to be of the form $x[n]$, then at any point n in the data set, this filter takes the average of the previous 50 points in $x[n]$.

- a) The moving average filter can be expressed as a 50-point filter. Call this filter $h[n]$ and express it in terms of the sum of time-shifted unit samples. Look at your lecture notes to see how we did this for a general signal.

- b) Go to the course website to download the file `problem_set3.txt`. Load this file into Matlab using the following command:

```
>> x=load('problem_set3.txt');
```

Create a plot showing `x`. This vector contains the measured irradiance over a single day (July 22nd, 2011).

Write a simple function that uses a for loop to produce the filtered signal $y[n] = h[n] * x[n]$.

Use the subplot command to show `x[n]`, `h[n]`, and `y[n]`. Use something like the following:

```
figure(1);  
subplot(3,1,1);  
stem(h);  
xlabel('n');  
ylabel('h[n]');  
subplot(3,1,2);  
plot(x);  
xlabel('n');  
ylabel('x[n]');  
subplot(3,1,3);  
plot(y);  
xlabel('n');  
ylabel('y[n]');
```

- c) Describe the differences between `x` and `y`. Clearly explain why this is a “moving average” filter using the definition of the convolution sum.