MEMORANDUM

DATE: January 22, 1998
TO: Jim Russell
FROM: Peter Tkacik
RE: Analysis of tube winding on ply widths, angles, and speeds
CC:

The goal of this report is to analyze the spiral tube winding process in order to help us better understand the relationship of product variation to manufacturing details. One of these might be inner tube ply gaps related to number of available ply widths.

Nomenclature of variables and diagram of tube:

To begin the analysis, a description of the tube being manufactured is made. This tube will have \( n \) number of plies and will be described using the following nomenclature:

\( n \) number of plies where the ply number \( i \) goes from \( i=1, 2, 3, \ldots, n \).

ID the inside diameter of the tube or the mandrel diameter.

OD the outside diameter of the tube.

\( t_i \) the thickness or caliper of the \( i^{th} \) ply of paper in the tube.

\( w_i \) the width of the \( i^{th} \) ply of paper in the tube.

\( d_i \) the diameter of the \( i^{th} \) ply of paper in the tube.

pitch the linear distance a tube travels with one revolution.

\( \alpha_i \) the angle of the \( i^{th} \) ply of paper into the tube with \( 0^\circ \) being convolute.

Speed the speed of the outside surface of the belt ply.

A visual description can be found in figure 1. as follows.
Figure 1. Tube making nomenclature
Geometric Analysis:

Since at the winder belt, all of the plies are moving in unison, they all share the pitch of the outer ply.

\[
\text{pitch} = \text{pitch}_1 = \text{pitch}_2 = \text{pitch}_3 = \ldots \text{pitch}_n
\]

eqn. 1

It should also be noted that the OD is the mandrel diameter plus the diametral thickness of the plies.

\[
\text{OD} = \text{ID} + 2[t_1 + t_2 + t_3 + \ldots + t_n].
\]

eqn. 2

which may be rewritten as...

\[
\text{OD} = \text{ID} + 2 \sum_{i=1, n} t_i.
\]

eqn. 3

For any intermediate ply \( i \), the diameter \( i \) is ...

\[
\text{diameter}_{i} = \text{ID} + 2 \sum_{k=1, i} t_k.
\]

eqn. 4

Now we should look at the belt ply or outer ply to calculate the pitch of the tube. The subscript for the outer ply is \( n \) so the diameter will be \( d_n \) and the angle will be \( \alpha_n \), etc. The ply angle for the outer ply (\( \alpha_n \)) is tied to the OD (\( d_n \)) and the belt ply width (width\( _n \)) by the geometry of the ply.

Assuming a butt joint on the outer ply, \( \alpha_n \) is calculated from the width and diameter by looking at a right triangle.

\[
\text{width}_n = \pi d_n \sin (\alpha_n)
\]

eqn. 5
or solving for \( \alpha_n \).

\[
\alpha_n = \sin^{-1}\left(\frac{\text{width}_n}{\pi d_n}\right)
\]

eqn. 6

The pitch can be then calculated from \( \alpha_n \), the \( \text{width}_n \) and the theory of similar triangles.

\[
\text{width}_n = \text{pitch} \cos (\alpha_n)
\]

Then we can solve for pitch.

\[
\text{pitch} = \frac{\text{width}_n}{\cos(\alpha_n)}
\]

eqn. 7

Now that we have calculated the pitch and the angle for the belt ply, we can calculate the under ply widths and angles, i.e. \( \text{width}_1 \) to \( \text{width}_{n-1} \) and \( \alpha_1 \) to \( \alpha_{n-1} \). Using a similar triangle from the same method as above we can calculate \( \alpha_i \).

\[
\tan(\alpha_i) = \frac{\text{pitch}}{\pi d_i}
\]

then we can solve for \( \alpha_i \).

\[
\alpha_i = \tan^{-1}\left(\frac{\text{pitch}}{\pi d_i}\right).
\]

eqn. 8

We can rewrite equation 5 with the variable \( i \) instead of the belt ply number \( n \) to let us solve for the intermediate ply width \( i \).

\[
\text{width}_i = \pi d_i \sin (\alpha_i)
\]

eqn. 9

Now that we have a relationship for the ply widths, angles, thickness’, and diameters, we may calculate the various ply speeds by looking at the length of ply wrap for each ply, i.e. the plylength \( i \). Since we know the pitch and the circumference for each ply, the plylength can be calculated using Pythagorean Theorem.
That is...

\[ \text{plylength}_i = \sqrt{\left(\pi d_i\right)^2 + \text{pitch}^2} \]  

eqn. 5

Since the winder speed is referenced to the belt ply or the winder belt...

\[ \text{Speed}_n = \text{Speed} \]  

eqn. 6

The speed of the lower plies is tied to the belt ply by the diametrical position.

\[ \text{speed}_i = \text{speed}_n \times \frac{\sqrt{\left(\pi d_i\right)^2 + \text{pitch}^2}}{\sqrt{\left(\pi d_n\right)^2 + \text{pitch}^2}} \]  

eqn. 7

**Tube Construction Rules:**

1. The general rule in Rock Hill is that we build up a core wall with 25 point paper for heavy walled cores of less than nine inch diameter. Using much thicker paper can result in formation and cracking problems and going below 25 point increases the number of plies which must be maintained and increases the chance for breakout.

2. The belt ply width is the starting point and is 5 7/16" wide.

3. The next ply down from the belt ply is 5 3/8" wide.

4. The plies below that drop as necessary in 1/8" increments.

5. The top ply is 14 point Kraft paper.

6. If the O.D. of the tube does not come up exactly to the specification, then the third or fourth from the belt ply may be exchanged for 30 point plies or a ply may be removed. These plies are usually mounted in a position which allows for quick replacement if the O.D. varies during a run.
Example Calculation: (660 wall paper mill core)

Construct a core which has an I.D. of Ø3.020" with a 0.660" wall, (resulting in a theoretical Ø4.340" O.D.).

Top Ply

To start the calculation, we will be working with the belt ply. It will be below the top ply which is 14 point Kraft paper. This results in the belt ply being formed at a wall of 0.660" - 0.014" = 0.646". Theoretically, this would require a belt ply formation diameter of 3.020" + 2×0.646" = 4.312". We may compensate for diametral variations when the tube OD is measured by the lasermike.

Belt Ply

As per the construction assumptions, the belt ply is $5\frac{7}{16}$ wide and is at a wall of 0.646". Since the top ply is at a diameter of 4.312", and since...

$$\alpha_n = \sin^{-1}\left(\frac{\text{width}_n}{\pi d_n}\right).$$

The ply angle is the arcsin of ( $5\frac{7}{16}$ / $\pi$ 4.312") = 23.7°.

Body Plies

The body plies are calculated the same way that the Belt Ply is calculated. The ply thickness’ are first developed changing 25 point for 30 point at appropriate places in order to reach the correct OD. Equation 8 is used to determine each new angle and equation 6 to determine the width for that angle. The theoretical ply gap is the difference between the zero ply gap ply widths and the actual ply widths (which step in increments of $\frac{1}{8}$).
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