MEMORANDUM

DATE: 21 September, 1999
TO: Jim Russell
FROM: Peter Tkacik
RE: Analysis of wide ply tube winding as compared to Konva Kore
CC: Larry McMillan

The goal of this report is to analyze the spiral tube winding process and in particular, how a 14” wide ply tube compares to a Konva Kore. In general, two things stand out:

1. The spiral winding speed of 173 bfm produces tubes at a rate of 13 tpm (a high Konva Kore speed).

2. The beam strength is reduced to 88% of Konva Kore levels (14% extra is needed to match KK levels).

Figure 1. Example wide ply spiral winder from 1965 patent
Appendix

Calculations and assumptions:

To begin the analysis, a description of the Konva Kore and wide ply tube being manufactured is made.

**Example case:**

- **ID**$_{KK}$ $\varnothing$4.00” (mandrel size)
- **W**$\text{wide}$ 14” is the width of the bottom ply of paper in the tube.
- **L**$_{KK}$ 150” is the tube length.
- **Speed**$_{KK}$ the speed of good running Konva Kore is 12 – 13 tpm (we will use 13).

**Nomenclature:**

- $n$ number of plies where the ply number $i$ goes from $i=1$, 2, 3, ..., $n$.
- **ID** the inside diameter of the tube or the mandrel diameter.
- **OD** the outside diameter of the tube.
- $t_i$ the thickness or caliper of the $i^{th}$ ply of paper in the tube.
- **W**$_i$ the width of the $i^{th}$ ply of paper in the tube.
- **d**$_i$ the diameter of the $i^{th}$ ply of paper in the tube.
- **pitch** the linear distance a tube travels with one revolution.
- **$\alpha_i$** the angle of the $i^{th}$ ply of paper into the tube with 0° being convolute.
- **Speed** the speed of the outside surface of the belt ply.

A visual description can be found in figure 2. as follows.
Figure 1. Tube making nomenclature

- Pitch
- Thickness
- Web width
- Ply = 1
- Ply = 2
- Ply = n
- \( \alpha_n \)
- \( \alpha_1 \)
- \( \text{Web width}_n \)
- \( \text{Web width}_1 \)
Geometric Analysis (General Case):

Since at the winder belt, all of the plies are moving in unison, they all share the pitch of the outer ply.

\[ pitch = pitch_1 = pitch_2 = pitch_3 = \ldots = pitch_n \]  
\[ \text{eqn. 1} \]

It should also be noted that the OD is the mandrel diameter plus the diametrical thickness of the plies.

\[ OD = ID + 2[t_1 + t_2 + t_3 + \ldots + t_n] \]  
\[ \text{eqn. 2} \]

which may be rewritten as...

\[ OD = ID + 2 \sum_{i=1,n} t_i \]  
\[ \text{eqn. 3} \]

For any intermediate ply \( i \), the diameter \( i \) is ...

\[ \text{diameter}_i = ID + 2 \sum_{k=1,i} t_k \]  
\[ \text{eqn. 4} \]

Now we should look at the belt ply or outer ply to calculate the pitch of the tube. The subscript for the outer ply is \( n \) so the diameter will be \( d_n \) and the angle will be \( \alpha_n \), etc. The ply angle for the outer ply \( (\alpha_n) \) is tied to the OD \( (d_n) \) and the belt ply width \( (\text{width}_n) \) by the geometry of the ply.

Assuming a butt joint on the outer ply, \( \alpha_n \) is calculated from the width and diameter by looking at a right triangle.

\[ \text{width}_n = \pi d_n \sin (\alpha_n) \]  
\[ \text{eqn. 5} \]
or solving for $\alpha_n$.

$$\alpha_n = \sin^{-1}\left(\frac{\text{width}_n}{\pi d_n}\right)$$  eqn. 6

The pitch can be then calculated from $\alpha_n$, the width$_n$ and the theory of similar triangles.

$$\text{width}_n = \text{pitch} \cos (\alpha_n)$$

Then we can solve for pitch.

$$\text{pitch} = \frac{\text{width}_n}{\cos(\alpha_n)}$$  eqn. 7

Now that we have calculated the pitch and the angle for the belt ply, we can calculate the under ply widths and angles, i.e. width$_1$ to width$_{n-1}$ and $\alpha_1$ to $\alpha_{n-1}$.

Using a similar triangle from the same method as above we can calculate $\alpha_i$.

$$\tan(\alpha_i) = \frac{\text{pitch}}{\pi d_i}$$

then we can solve for $\alpha_i$.

$$\alpha_i = \tan^{-1}\left(\frac{\text{pitch}}{\pi d_i}\right).$$  eqn. 8

We can rewrite equation 5 with the variable $i$ instead of the belt ply number $n$ to let us solve for the intermediate ply width$_i$.

$$\text{width}_i = \pi d_i \sin (\alpha_i)$$  eqn. 9

Now that we have a relationship for the ply widths, angles, thickness’, and diameters, we may calculate the various ply speeds by looking at the length of ply wrap for each ply, i.e. the plylength$_i$. Since we know the pitch and the circumference for each ply, the plylength can be calculated using Pythagorean Theorem.
That is...

\[ \text{plylength}_i = \frac{\sqrt{\left(\pi d_i\right)^2 + \text{pitch}^2}}{\text{pitch}} \]

Since the winder speed is referenced to the belt ply or the winder belt...

\[ \text{Speed}_n = \text{Speed} \]

The speed of the lower plies is tied to the belt ply by the diametrical position.

\[ \text{speed}_i = \text{speed}_n \times \frac{\sqrt{\left(\pi d_i\right)^2 + \text{pitch}^2}}{\sqrt{\left(\pi d_n\right)^2 + \text{pitch}^2}}. \]

**Tube Construction Rules:**

1. The general rule in Rock Hill is that we *build up a core wall with 25 point paper* for heavy walled cores of less than nine-inch diameter. Using much thicker paper can result in formation and cracking problems and going below 25 point increases the number of plies which must be maintained and increases the chance for breakout.

2. *The belt ply width is the starting point and for this study is 14” wide.*

3. *The plies below that drop as necessary in \( \frac{1}{8}’’ \) increments.*

4. If the O.D. of the tube does not come up exactly to the specification, then *the third or fourth from the belt ply may be exchanged for 30 point plies or a ply may be removed.* These plies are usually mounted in a position, which allows for quick replacement if the O.D. varies during a run.
Example Calculation: (0.375” wall wide ply carpet core)

Construct a core which has an I.D. of Ø4.000” with a 0.375” wall, (resulting in a theoretical Ø4.750” O.D.) with a 14” top ply.

Top Ply

To start the calculation, we will be working with the belt ply. As per the construction assumptions, the belt ply is 14” wide and is at a wall of 0.375”. Since the top ply is at a diameter of 4.750”, and since...

\[ \alpha_n = \sin^{-1}\left(\frac{\text{width}_n}{\pi d_n}\right). \]

Eqn. 8

The ply angle is the arcsin of \((14” / \pi 4.750”) = 69.74^\circ\).

Body Plies

The body plies are calculated the same way that the Belt Ply is calculated. The ply thicknesses are first developed (changing 25 point for 30 point at appropriate places in order to reach the correct OD). Equation 8 is used to determine each new angle and equation 6 to determine the width for that angle. The theoretical ply gap is the difference between the zero ply gap ply widths and the actual ply widths (which step in increments of \(\frac{1}{8}”\)).

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<th>Ply gap</th>
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**Speed Analysis:**

Since at the winder belt, all of the plies are moving in unison, they all share the pitch of the outer ply.

\[ \text{pitch} = \text{pitch}_1 = \text{pitch}_2 = \text{pitch}_3 = \ldots \text{pitch}_n \quad \text{eqn. 1} \]

\[ \text{pitch} = \frac{\text{width}}{\cos(\alpha_n)} = \frac{14}{\cos(69.74^\circ)} = 40.43'' \quad \text{eqn. 9} \]

\[ \text{plylength}_i = \sqrt{(\pi d_i)^2 + \text{pitch}^2} = \sqrt{(\pi 4.75)^2 + 40.43^2} = 43.10 \quad \text{eqn. 5} \]

Since the winder speed is referenced to the belt ply or the winder belt...

\[ \text{Speed}_n = \text{Speed} \quad \text{eqn. 10} \]

The speed of the lower plies is tied to the belt ply by the diametrical position. For the Konva Kore, the speed in interrupted and averages to produce 13 tpm at 150” each. To match this with a spiral machine, the tube would have to have a linear speed of

\[ 150”/\text{tube} \times 13\text{tubes/min} \div 12”/\text{ft} = 162.5 \text{ feet/min} \quad \text{eqn. 11} \]

To match this on a spiral winder at a spiral-winding angle of 69.75°, you would have to run at a belt speed of

\[ \text{Belt speed} = \text{Linear speed} \times \text{plylength} / \text{pitch} \quad \text{eqn. 12} \]

\[ \text{Belt speed} = 162.5 \times 43.10 / 40.43 = 173.2 \text{ bfm.} \]

A wide ply spiral winder would need to run 173 bfm in order to match a 13 tpm Konva Kore machine.
Beam Strength Analysis:

The beam strength of a spiral tube is reduced due to the angle of the fibers away from the longitudinal direction. For cylinder board, the MD to CD strength ratio is assumed to be...

\[
\frac{\text{Strength}_{MD}}{\text{Strength}_{CD\_cylinder}} = \frac{65}{35} \quad \text{eqn. 13}
\]

If the plies are oriented in a diagonal pattern 69.74° from longitudinal, then the effective strength can be solved for using the equation of an ellipse.

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{equation of an ellipse} \quad \text{eqn. 14}
\]

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 = \frac{x^2}{65^2} + \frac{y^2}{35^2} \quad \text{eqn. 15}
\]

The ratio of \(x\) to \(y\) is...

\[
\frac{x}{y} = \tan(69.74^\circ) \quad \text{eqn. 16}
\]

\[y = \frac{x}{\tan(69.74^\circ)} = 0.369x \quad \text{eqn. 17}\]

Simultaneously solving equations 15 and 17 for \(x\) results in...

\[
\frac{x^2}{65^2} + \left(\frac{x}{\tan(69.74^\circ)}\right)^2 = 1 \quad \text{eqn. 18}
\]

\[
1 = \frac{x^2}{65^2} + \left(\frac{0.369x}{35^2}\right)^2 = x^2\left(\frac{1}{65^2} + \frac{0.1362}{35^2}\right) = 0.000348 \quad \text{eqn. 19}
\]
\[ x^2 = \frac{1}{0.000348} = 2874 \quad \text{eqn. 20} \]

\[ x = 53.61 \quad \text{eqn. 21} \]

Solving for \( y \) gives...

\[ y = \frac{x}{\tan(69.74^\circ)} = 19.79 \quad \text{eqn. 22} \]

The strength in the 69.74° direction is the square root of the sum of the squares

\[
\sqrt{x^2 + y^2} = \sqrt{19.79^2 + 53.61^2} = 57.15
\quad \text{eqn. 23}
\]

The reduction from the convolute wrapped core is the ratio of the two...

\[
\frac{\text{WidePlyStrength}}{\text{ConvoluteStrength}} = \frac{57.15}{65} = 0.879 = 87.9\% \approx 88\%
\quad \text{eqn. 24}
\]

Therefore, to match the beam strength of the Konva Kore tube, the wide ply tube would have to add material.

\[
\frac{1}{87.9\%} = 1.138
\quad \text{eqn. 25}
\]

An increase of...

\[ 1.138 - 1 = 0.138 = 13.8\% \approx 14\% \quad \text{eqn. 26} \]

14% more material strength is needed in a wide ply tube to match the beam strength of a Konva Kore tube.