Chapter 5

Transformer Design Trade-Offs

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Introduction

The conversion process in power electronics requires the use of transformer components that are frequently the heaviest and bulkiest item in the conversion circuit. They also have a significant effect upon the overall performance and efficiency of the system. Accordingly, the design of such transformers has an important influence on the overall system weight, power conversion efficiency, and cost. Because of the interdependence and interaction of these parameters, judicious trade-offs are necessary to achieve design optimization.

The Design Problem Generally

The designer is faced with a set of constraints that must be observed in the design on any transformer. One of these constraints is the output power, P_o (operating voltage multiplied by maximum current demand) in that the secondary winding must be capable of delivering to the load within specified regulation limits. Another constraint relates to minimum efficiency of operation, which is dependent upon the maximum power loss that can be allowed in the transformer. Still another constraint defines the maximum permissible temperature rise for the transformer when it is used in a specified temperature environment.

One of the basic steps in transformer design is the selection of proper core material. Magnetic materials used to design low and high frequency transformers are shown in Table 5-1. Each one of these materials has its own optimum point in the cost, size, frequency and efficiency spectrum. The designer should be aware of the cost difference between silicon-iron, nickel-iron, amorphous and ferrite materials. Other constraints relate to volume occupied by the transformer and, particularly in aerospace applications, weight minimization is an important goal. Finally, cost effectiveness is always an important consideration.

Depending upon the application, some of these constraints will dominate. Parameters affecting others may then be traded off, as necessary, to achieve the most desirable design. It is not possible to optimize all parameters in a single design because of their interaction and interdependence. For example, if volume and weight are of great significance, reductions in both can often be affected, by operating the transformer at a higher frequency, but with the penalty being in efficiency. When, the frequency cannot be increased, reduction in weight and volume may still be possible by selecting a more efficient core material, but with the penalty of increased cost. Thus, judicious trade-offs must be affected to achieve the design goals.

Transformer designers have used various approaches in arriving at suitable designs. For example, in many cases, a rule of thumb is used for dealing with current density. Typically, an assumption is made that a good working level is 200 amps-per-cm^2 (1000 circular mils-per-ampere). This rule of thumb will work in many instances, but the wire size needed to meet this requirement may produce a heavier and bulkier transformer than desired or required. The information presented in this Chapter makes it possible to avoid the assumption use of this and other rules of thumb, and to develop a more economical design with great accuracy.

Table 5-1. Magnetic Materials and Their Characteristics

Magnetic Core Material Characteristics					
Material Name	Initial Permeability μ_i	Flux Density Tesla B_s	Curie Temperature °C	dc, Coercive Force, Hc Oersteds	Operating Frequency f
Iron Alloys					
Magnesil	1.5 K	1.5-1.8	750	0.4-0.6	< 2kHz
Supermendur*	0.8 K	1.9-2.2	940	0.15-0.35	< 1kHz
Orthonol	2 K	1.42-1.58	500	0.1-0.2	< 2kHz
Sq.Permalloy	12 K-100 K	0.66-0.82	460	0.02-0.04	< 25kHz
Supermalloy	10 K-50 K	0.65-0.82	460	0.003-0.008	< 25kHz
Amorphous					
2605-SC	3K	1.5-1.6	370	0.03-0.08	< 250kHz
2714A	20K	0.5-0.58	> 200	0.008-0.02	< 250kHz
Vitro perm 500	30K	1.0-1.2	> 200	< 0.05	< 250kHz
Ferrite					
MnZn	0.75-15K	0.3-0.5	100-300	0.04-0.25	< 2MHz
NiZn	15-1500	0.3-0.5	150-450	0.3-0.5	< 100MHz
* Field Anneal.					

Power Handling Ability

For years, manufacturers have assigned numeric codes to their cores to indicate their power-handling ability. This method assigns to each core a number called the area product, A_p. That is the product of the window area, W_a, and the core cross-section, A_c. The core suppliers use these numbers to summarize dimensional and electrical properties in their catalogs. They are available for laminations, C cores, ferrite cores, powder cores, and toroidal tape wound cores.

Relationship, A_p, to Transformer Power Handling Capability

Transformers

According to the newly developed approach, the power handling capability of a core is related to its area product, A_p, by an equation, which may be stated as:

$$A_p = \frac{P_t\left(10^4\right)}{K_f K_u B_m J f}, \quad [\text{cm}^4] \qquad [5\text{-}1]$$

Where:

Kf = waveform coefficient

4.0 square wave
4.44 sine wave

From the above, it can be seen that factors such as flux density, frequency of operation, and window utilization factor K_u, define the maximum space which may be occupied by the copper in the window.

Relationship, K_g, to Transformer Regulation and Power Handling Capability

Although most transformers are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and power-handling ability of a core is related to two constants:

$$\alpha = \frac{P_t}{2K_g K_e}, \quad [\%] \qquad [5\text{-}2]$$

α = Regulation (%) [5-3]

The constant, K_g, (See Chapter 7) is determined by the core geometry, which may be related by the following equations:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \qquad [5\text{-}4]$$

The constant, K_e, is determined by the magnetic and electric operating conditions, which may be related by the following equation:

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right) \qquad [5\text{-}5]$$

Where:

Kf = waveform coefficient

4.0 square wave
4.44 sine wave

From the above, it can be seen that factors such as flux density, frequency of operation, and waveform coefficient, have an influence on the transformer size. Because of their significance, the area product, A_p, and the core geometry, K_g, are treated extensively in this handbook. A great deal of other information is also presented for the convenience of the designer. Much of the information is in tabular form to assist designers in making the trade-offs best suited for the particular application, in a minimum amount of time.

Transformer Area Product, A_p

The author has developed additional relationships between, A_p, numbers and current density, J, for given regulation and temperature rise. The area product, A_p, is a length dimension to the fourth power, (l^4), as shown in Figure 5-1.

$$W_a = FG, \quad [\text{cm}^2]$$
$$A_c = DE, \quad [\text{cm}^2] \qquad [5\text{-}6]$$
$$A_p = W_a A_c, \quad [\text{cm}^4]$$

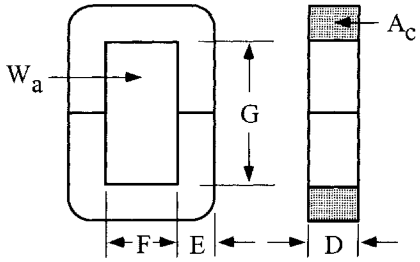

Figure 5-1. C Core Outline Showing the Window Area, W_a and Iron Area, A_c.

It should be noted. The constants for tape-wound cores, such as: K_{vol}, K_w, K_s, K_j and K_p will have a tendency to jump around and not be consistent. This inconsistency has to do with the core being in a housing, without true proportions.

Transformer Volume and the Area Product, A_p

The volume of a transformer can be related to the area product, A_p of a transformer, treating the volume, as shown in Figures 5-2 to 5-4, as a solid quantity without any subtraction for the core window. The relationship is derived according to the following reasoning: Volume varies in accordance with the cube of any linear dimension, (l), whereas area product, A_p, varies as the fourth power:

$$\text{Volume} = K_1 l^3, \quad [\text{cm}^3] \quad [5\text{-}7]$$

$$A_p = K_2 l^4, \quad [\text{cm}^4] \quad [5\text{-}8]$$

$$l^4 = \frac{A_p}{K_2} \quad [5\text{-}9]$$

$$l = \left(\frac{A_p}{K_2}\right)^{(0.25)} \quad [5\text{-}10]$$

$$l^3 = \left[\left(\frac{A_p}{K_2}\right)^{0.25}\right]^3 = \left(\frac{A_p}{K_2}\right)^{0.75} \quad [5\text{-}11]$$

$$\text{Volume} = K_1\left(\frac{A_p}{K_2}\right)^{0.75} \quad [5\text{-}12]$$

$$K_{vol} = \frac{K_1}{K_2^{(0.75)}} \quad [5\text{-}13]$$

The volume-area product, A_p, relationship is therefore:

$$\text{Volume} = K_{vol}\,A_p^{(0.75)}, \quad [\text{cm}^3] \quad [5\text{-}14]$$

in which, K_{vol}, is a constant related to core configuration whose values are given in Table 5-2. These values were obtained by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-2. Volume-Area Product Relationship.

Volume-Area Product Relationship	
Core Type	K_{vol}
Pot Core	14.5
Powder Core	13.1
Laminations	19.7
C Core	17.9
Single-coil C Core	25.6
Tape-wound Core	25.0

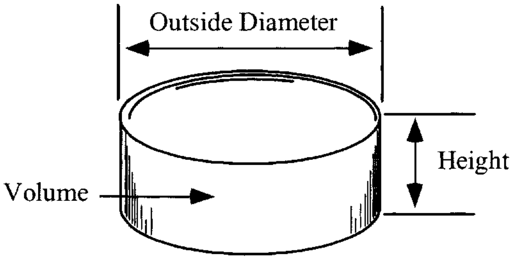

Figure 5-2. Toroidal Transformer Outline, Showing the Volume.

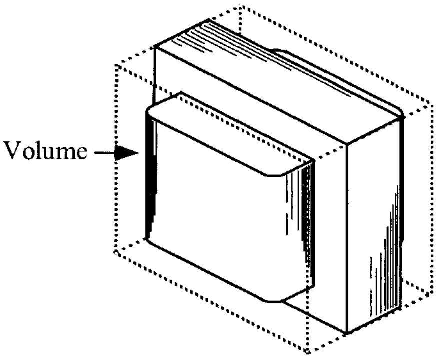

Figure 5-3. EI Core Transformer Outline, Showing the Volume.

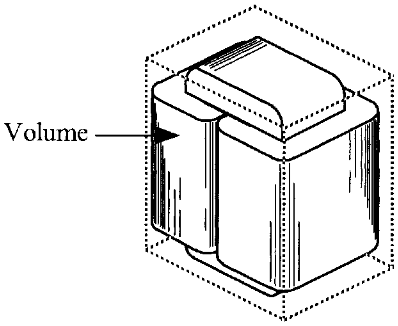

Figure 5-4. C Core Transformer Outline, Showing the Volume.

The relationship between volume and area product, A_p, for various core types is graphed in Figures 5-5 through 5-7. The data for these Figures has been taken from Tables in Chapter 3.

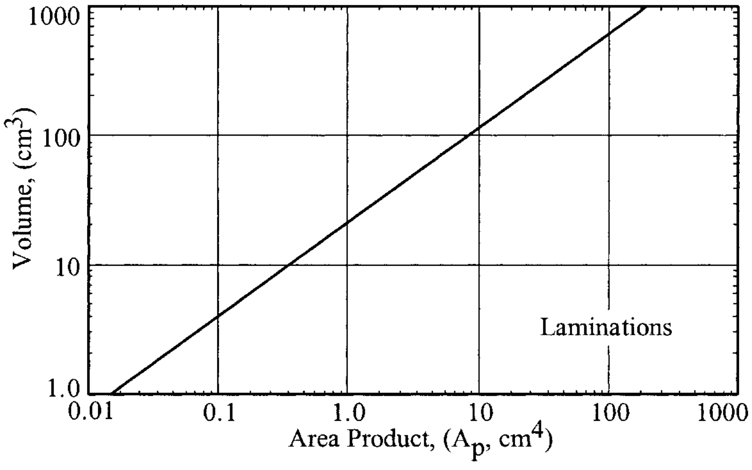

Figure 5-5. Volume Versus Area Product, A_p for EI Laminations.

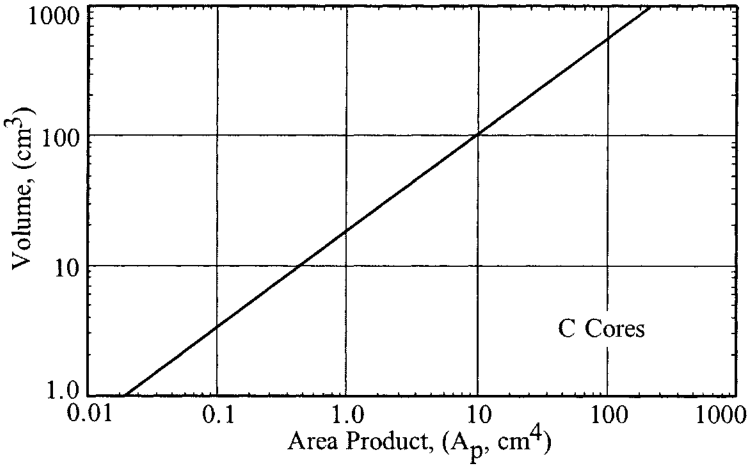

Figure 5-6. Volume Versus Area Product, A_p for C Cores.

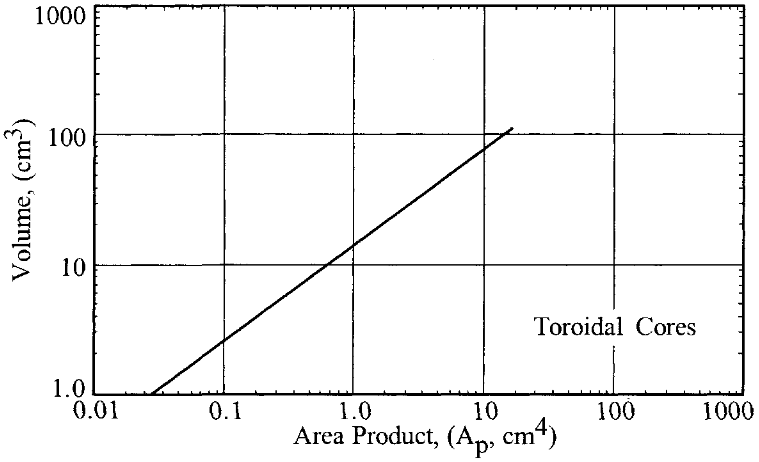

Figure 5-7. Volume Versus Area Product, A_p, for Toroidal MPP Cores.

Transformer Weight and the Area Product, A_p

The total weight of a transformer can also be related to the area product, A_p, of a transformer. The relationship is derived according to the following reasoning: weight, W_t, varies, in accordance with the cube of any linear dimension l, whereas area product, A_p, varies, as the fourth power:

$$W_t = K_3 l^3, \quad \text{[grams]} \quad [5\text{-}15]$$

$$A_p = K_2 l^4, \quad [\text{cm}^4] \quad [5\text{-}16]$$

$$l^4 = \frac{A_p}{K_2} \quad [5\text{-}17]$$

$$l^4 = \left(\frac{A_p}{K_2}\right)^{(0.25)} \quad [5\text{-}18]$$

$$l^3 = \left[\left(\frac{A_p}{K_2}\right)^{0.25}\right]^3 = \left(\frac{A_p}{K_2}\right)^{0.75} \quad [5\text{-}19]$$

$$W_t = K_3 \left(\frac{A_p}{K_2}\right)^{0.75} \quad [5\text{-}20]$$

$$K_w = \frac{K_3}{K_2^{(0.75)}} \quad [5\text{-}21]$$

The weight-area product, A_p, relationship is therefore:

$$W_t = K_w A_p^{(0.75)} \quad [5\text{-}22]$$

in which, K_w, is a constant related to core configuration, whose values are given in Table 5-3, These values were obtained by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-3. Weight-Area Product Relationship.

Weight-Area Product Relationship	
Core Type	K_w
Pot Core	48.0
Powder Core	58.8
Laminations	68.2
C Core	66.6
Single-coil C Core	76.6
Tape-wound Core	82.3

The relationship between weight and area product, A_p, for various core types is graphed in Figures 5-8 through 5-10. The data for Figures 5-8 through 5-10 has been taken from Tables in Chapter 3.

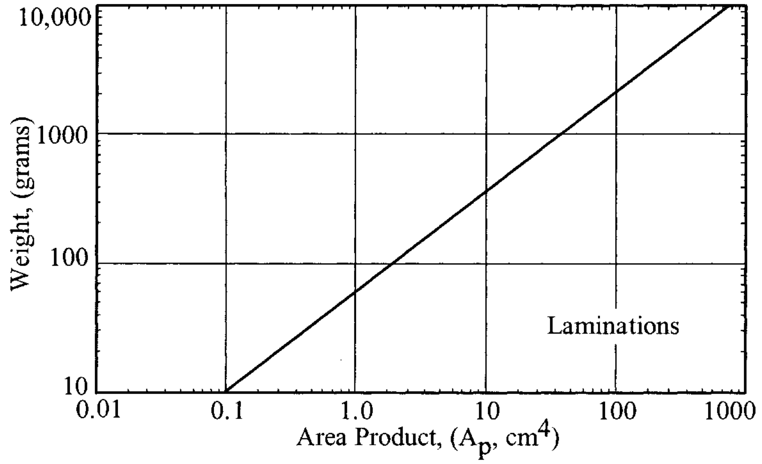

Figure 5-8. Total Weight Versus Area Product, A_p, for EI Laminations.

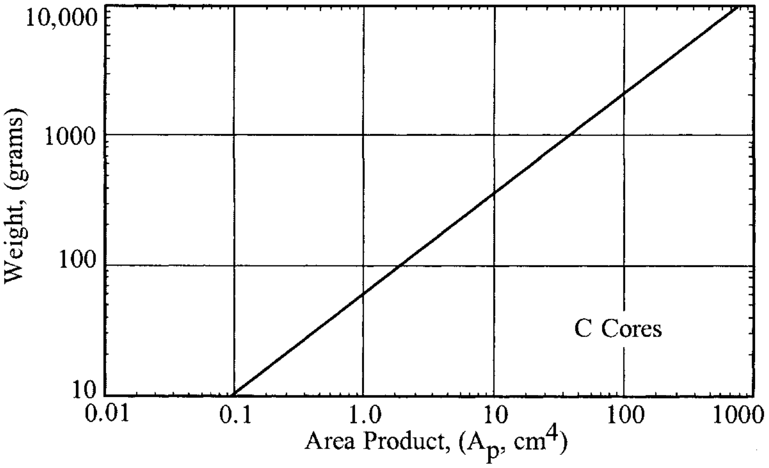

Figure 5-9. Total Weight Versus Area Product, A_p, for C Cores.

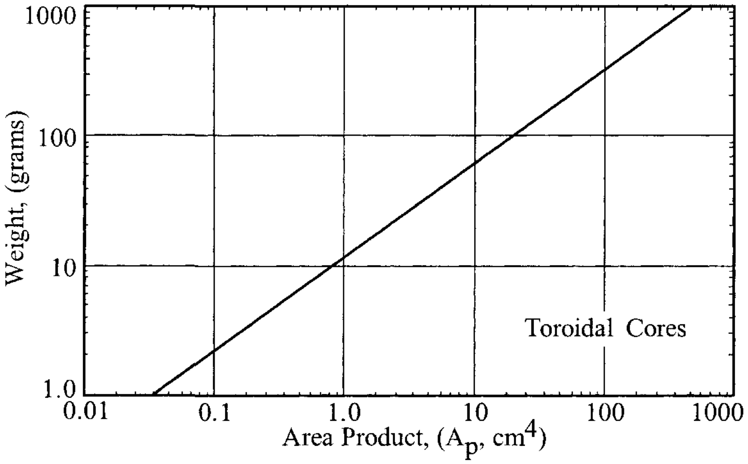

Figure 5-10. Total Weight Versus Area Product, A_p, for Toroidal MPP Cores.

Transformer Surface Area and the Area Product, A_p

The surface area of a transformer can be related to the area product, A_p, of a transformer, treating the surface area, as shown in Figure 5-11 through 5-13. The relationship is derived in accordance with the following reasoning: the surface area varies with the square of any linear dimension (l), whereas the area product, A_p, varies as the fourth power.

$$A_t = K_4 l^2, \quad [\text{cm}^2] \quad [5\text{-}23]$$

$$A_p = K_2 l^4, \quad [\text{cm}^4] \quad [5\text{-}24]$$

$$l^4 = \frac{A_p}{K_2} \quad [5\text{-}25]$$

$$l = \left(\frac{A_p}{K_2}\right)^{(0.25)} \quad [5\text{-}26]$$

$$l^2 = \left[\left(\frac{A_p}{K_2}\right)^{0.25}\right]^2 = \left(\frac{A_p}{K_2}\right)^{0.5} \quad [5\text{-}27]$$

$$A_t = K_4 \left(\frac{A_p}{K_2}\right)^{0.5} \quad [5\text{-}28]$$

$$K_s = \frac{K_4}{K_2^{(0.5)}} \quad [5\text{-}29]$$

The relationship between surface area, A_t and area product, A_p can be expressed as:

$$A_t = K_s A_p^{(0.5)} \quad [5\text{-}30]$$

in which, K_s, is a constant related to core configuration, whose values are given in Table 5-4. These values were obtained by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-4. Surface Area-Area Product Relationship.

Surface Area-Area Product Relationship	
Core Type	K_s
Pot Core	33.8
Powder Core	32.5
Laminations	41.3
C Core	39.2
Single-coil C Core	44.5
Tape-wound Core	50.9

The surface area for toroidal type transformers is calculated, as shown below.

$$\text{Top and Bottom Surface} = 2\left(\frac{\pi(OD)^2}{4}\right), \quad [\text{cm}^2]$$

$$\text{Periphery Surface} = \left(\pi(OD)\right)(\text{Height}), \quad [\text{cm}^2] \quad [5\text{-}31]$$

$$A_t = \frac{\pi(OD)^2}{2} + (\pi(OD)(\text{Height}), \quad [\text{cm}^2]$$

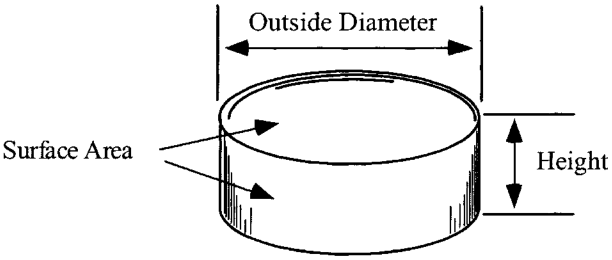

Figure 5-11. Toroidal Transformer Outline Showing the Surface Area.

The surface areas for C cores, Laminations and similar configurations are calculated as shown below. There is a small amount of area that is deducted because the sides and the ends are not a complete square.

$$\begin{aligned} &\text{End} = (\text{Height})(\text{Length}), \quad [\text{cm}^2] \\ &\text{Top} = (\text{Length})(\text{Width}), \quad [\text{cm}^2] \\ &\text{Side} = (\text{Height})(\text{Width}), \quad [\text{cm}^2] \\ &\text{Surface Area} = 2(\text{End}) + 2(\text{Top}) + 2(\text{Side}), \quad [\text{cm}^2] \end{aligned} \quad [5\text{-}32]$$

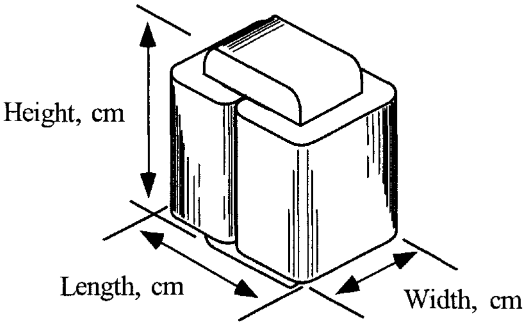

Figure 5-12. C Core Transformer Outline, Showing the Surface Area.

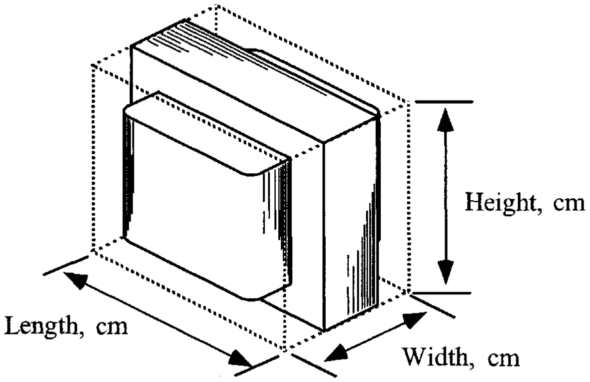

Figure 5-13. Typical EE or EI Transformer Outline, Showing the Surface Area.

The relationship between surface area and area product, A_p, for various core types is graphed in Figures 5-14 through 5-16. The data for these Figures has been taken from Tables in Chapter 3.

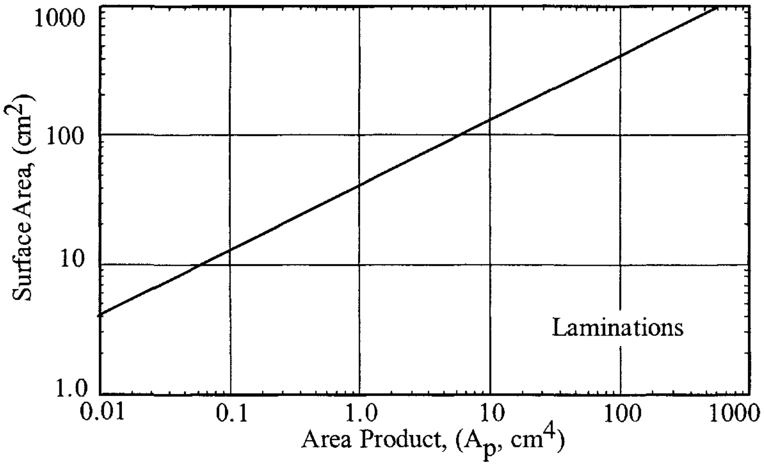

Figure 5-14. Surface Area, A_t, Versus Area Product, A_p, for EI Laminations.

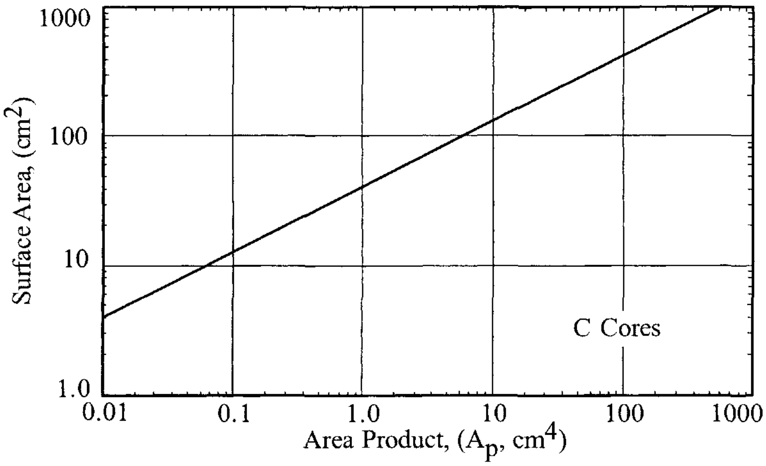

Figure 5-15. Surface Area, A_t, Versus Area Product, A_p, for C Cores.

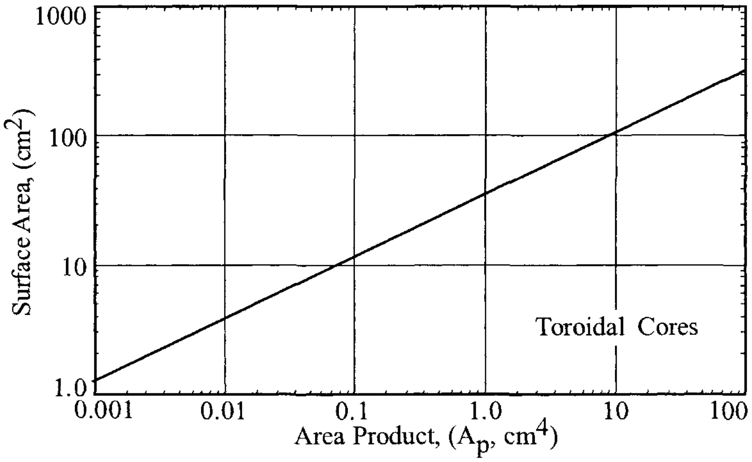

Figure 5-16. Surface Area, A_t, Versus Area Product, A_p, for Toroidal MPP Cores.

Transformer Current Density, J, and the Area Product, A_p

The current density, J, of a transformer can be related to the area product, A_p, of a transformer for a given temperature rise. The relationship can be derived as follows:

$$A_t = K_s A_p^{(0.5)}, \quad [\text{cm}^2] \quad [5\text{-}33]$$

$$P_{cu} = I^2 R, \quad [\text{watts}] \quad [5\text{-}34]$$

$$I = A_w J, \quad [\text{amps}] \quad [5\text{-}35]$$

Therefore,

$$P_{cu} = A_w^2 J^2 R \quad [5\text{-}36]$$

And since,

$$R = \frac{\text{MLT}}{A_w} N\rho, \quad [\text{ohms}] \quad [5\text{-}37]$$

We have:

$$P_{cu} = A_w^2 J^2 \frac{\text{MLT}}{A_w} N\rho \quad [5\text{-}38]$$

$$P_{cu} = A_w J^2 (\text{MLT}) N\rho \quad [5\text{-}39]$$

Since MLT has a dimension of length,

$$\text{MLT} = K_5 A_p^{(0.25)} \quad [5\text{-}40]$$

$$P_{cu} = A_w J^2 \left(K_5 A_p^{(0.25)}\right) N\rho \quad [5\text{-}41]$$

$$A_w N = K_3 W_a = K_6 A_p^{(0.5)} \quad [5\text{-}42]$$

$$P_{cu} = \left(K_6 A_p^{(0.5)}\right)\left(K_5 A_p^{(0.25)}\right) J^2 \rho \quad [5\text{-}43]$$

Let:

$$K_7 = K_6 K_5 \rho \quad [5\text{-}44]$$

Then assuming the core loss is the same as the copper loss for optimized transformer operation: (See Chapter 6),

$$P_{cu} = K_7 A_p^{(0.75)} J^2 = P_{fe} \quad [5\text{-}45]$$

$$P_\Sigma = P_{cu} + P_{fe} \quad [5\text{-}46]$$

$$\Delta T = K_8 \frac{P_\Sigma}{A_t} \quad [5\text{-}47]$$

$$\Delta T = \frac{2 K_8 K_7 J^2 A_p^{(0.75)}}{K_s A_p^{(0.5)}} \quad [5\text{-}48]$$

To simplify, let:

$$K_9 = \frac{2 K_8 K_7}{K_s} \quad [5\text{-}49]$$

Then,

$$\Delta T = K_9 J^2 A_p^{(0.25)} \quad [5\text{-}50]$$

$$J^2 = \frac{\Delta T}{K_9 A_p^{(0.25)}} \quad [5\text{-}51]$$

Then, letting:

$$K_{10} = \frac{\Delta T}{K_9} \quad [5\text{-}52]$$

We have:

$$J^2 = K_{10} A_p^{(0.25)} \quad [5\text{-}53]$$

The relationship between current density, J, and area product, A_p, can, therefore, be expressed as:

$$J = K_j A_p^{(0.125)} \quad [5\text{-}54]$$

The constant, K_j, is related to the core configuration, whose values are given in Table 5-5. These values have been derived by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-5. Constant, K_j, for Temperature Increase of 25°C and 50°C.

Temperature Constant, K_j		
Core Type	K_j (Δ25°)	K_j (Δ50°)
Pot Core	433	632
Powder Core	403	590
Laminations	366	534
C Core	322	468
Single-coil C Core	395	569
Tape-wound Core	250	365

The relationship between current density, J, and area product, A_p, for temperature increases of 25°C and 50°C is graphed in Figures 5-17 through 5-19 from data calculated of Tables 3-1 through 3-64 in Chapter 3.

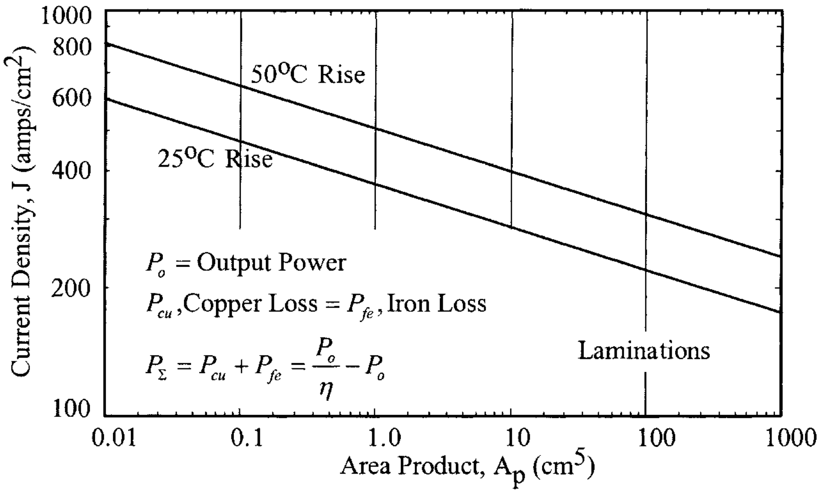

Figure 5-17. Current Density, J, Versus Area Product, A_p, for EI Laminations.

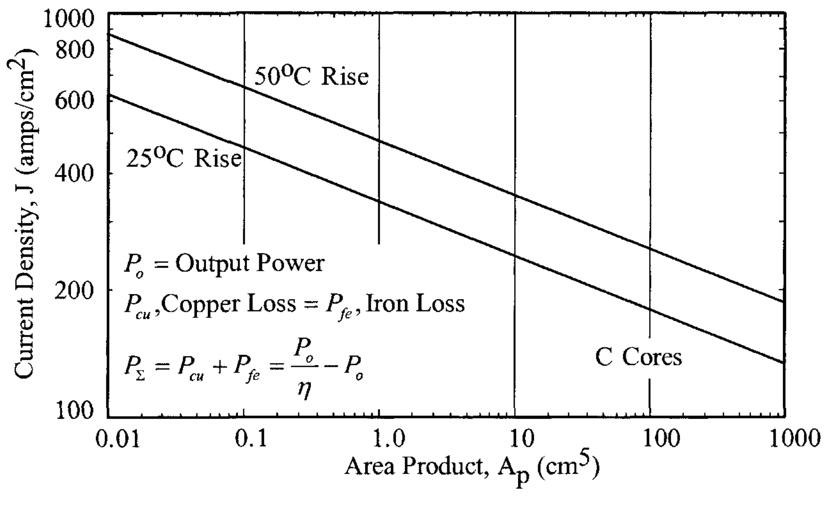

Figure 5-18. Current Density, J, Versus Area Product, A_p, for C Cores.

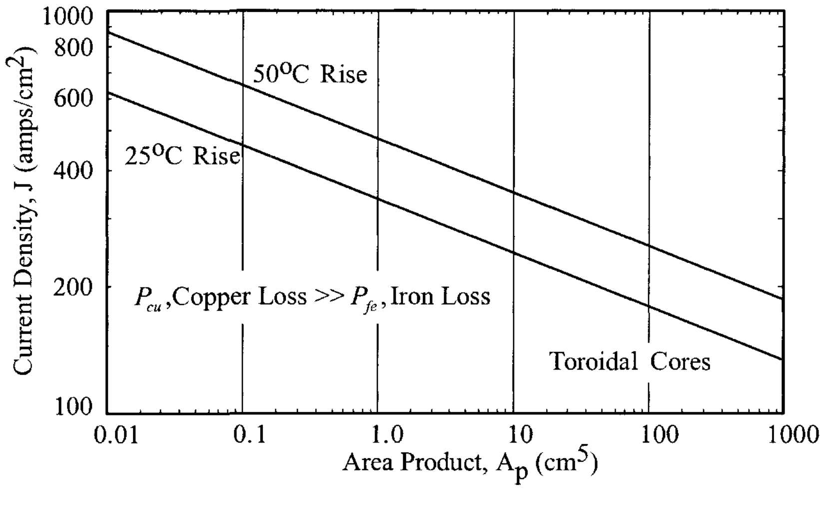

Figure 5-19. Current Density, J, Versus Area Product, A_p, for MPP Cores.

Transformer Core Geometry, K_g, and the Area Product, A_p

The core geometry, K_g, of a transformer can be related to the area product, A_p. The relationship is according to the following: the core geometry, K_g, varies in accordance with the fifth power of any linear dimension, (1), whereas area product, A_p, varies as the fourth power.

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5] \quad [5\text{-}55]$$

$$K_g = K_{10} l^5 \quad [5\text{-}56]$$

$$A_p = K_2 l^4 \quad [5\text{-}57]$$

From Equation 5-56,

$$l = \left(\frac{K_g}{K_{10}}\right)^{(0.2)} \quad [5\text{-}58]$$

Then,

$$l^4 = \left[\left(\frac{K_g}{K_{10}}\right)^{(0.2)}\right]^4 = \left(\frac{K_g}{K_{10}}\right)^{(0.8)} \quad [5\text{-}59]$$

Substituting Equation 5-59 into Equation 5-57,

$$A_p = K_2\left(\frac{K_g}{K_{10}}\right)^{(0.8)} \quad [5\text{-}60]$$

Let:

$$K_p = \frac{K_2}{K_{10}^{(0.8)}} \quad [5\text{-}61]$$

Then,

$$A_p = K_p K_g^{(0.8)} \quad [5\text{-}62]$$

The constant, K_p, is related to the core configuration, whose values are given in Table 5-6. These values have been derived by averaging the values from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

Table 5-6. Configuration Constant, K_p, for Area Product, A_p, and Core geometry, K_g.

Constant, K_p	
Core Type	K_p
Pot Core	8.9
Powder Core	11.8
Laminations	8.3
C Core	12.5
Tape-wound Core	14.0

The relationship between area product, A_p, and core geometry, K_g, is graphed in Figures 5-20 through 5-22, from the data taken from Tables 3-1 through Tables 3-64 in Chapter 3.

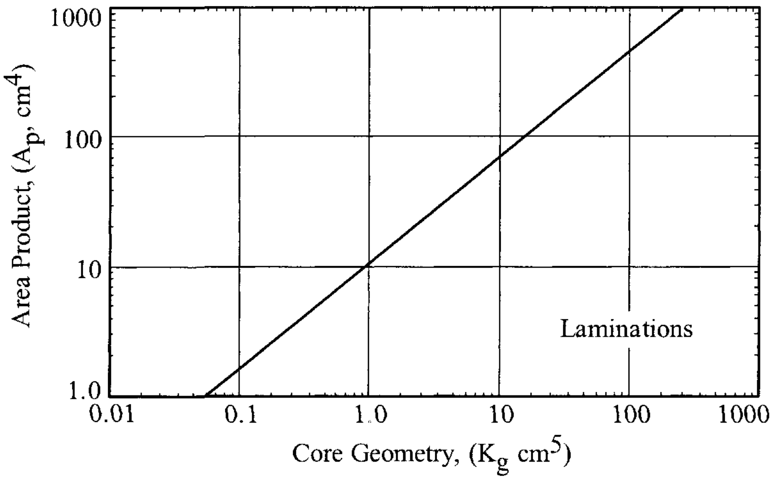

Figure 5-20. Area Product, A_p, Versus Core Geometry, K_g, for EI Laminations.

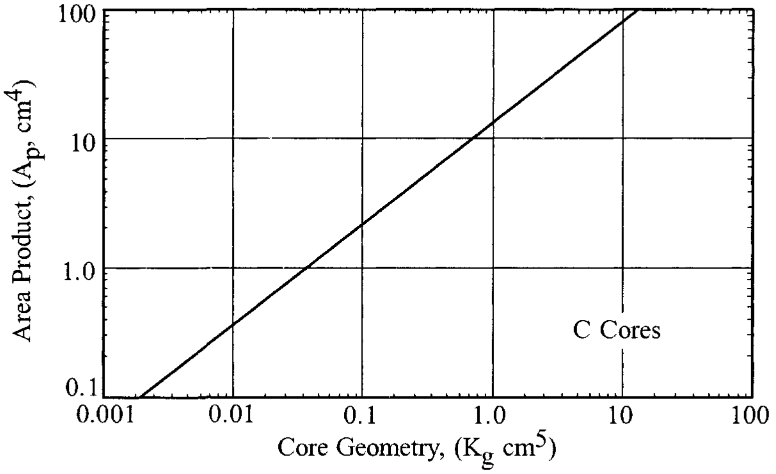

Figure 5-21. Area Product, A_p, Versus Core Geometry, K_g, for C Cores.

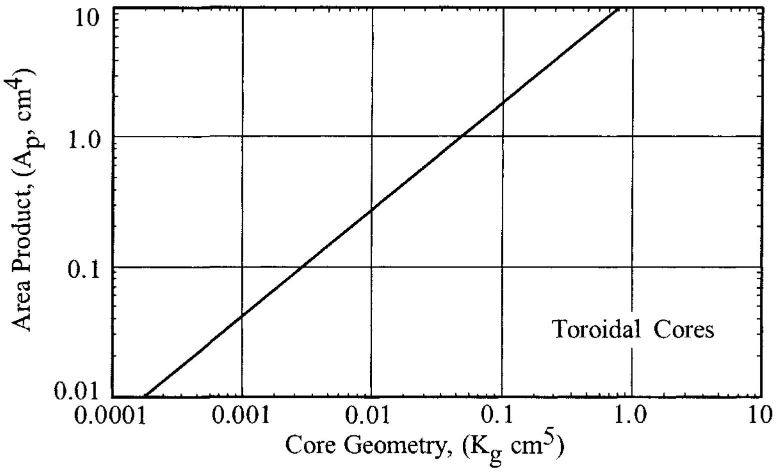

Figure 5-22. Area Product, A_p, Versus Core Geometry, K_g, for MPP Powder Cores.

Weight Versus Transformer Regulation

There are many design tasks where the transformer weight is very important in meeting the design specification. The engineer will raise the operating frequency in order to reduce the size and weight. The magnetic materials will be reviewed for performance at the operating frequency and at minimum and maximum temperatures. When the idealized magnetic material has been found and the weight of the transformer is still too high, then the only solution is to change the regulation. The regulation of a transformer versus the weight is shown in Figure 5-23. There are times when the engineer would like to know what the weight impact would be, if the regulation were to be increased or decreased.

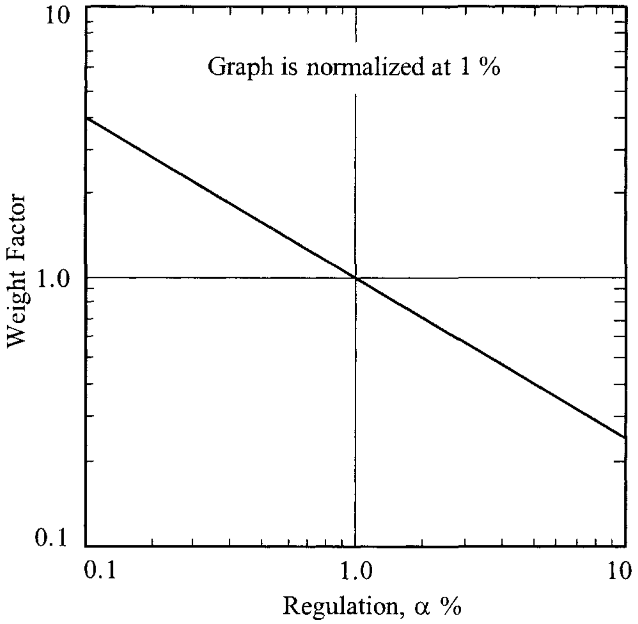

Figure 5-23. Weight Versus Regulation.

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