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Introduction

Current transformers are used to measure, or monitor, the current in the lead of an ac power circuit. They are very useful in high-power circuits, where the current is large, i.e., higher than the ratings of so-called self-contained current meters. Other applications relate to overcurrent and undercurrent relaying for power circuit protection, such as, in the power lead of an inverter or converter. Multiturn secondaries then provide a reduced current for detecting overcurrent, undercurrent, peak current, and average current, as shown in Figure 16-1.

![Figure 16-1. Simple, Secondary AC Current Monitor.](image)

In current transformer designs, the core characteristics must be carefully selected because excitation current, $I_{n}$ essentially subtracts from the metered current and effects the true ratio and phase angle of the output current.

The simplified equivalent circuit of a current transformer, as shown in Figure 16-2, represents the important elements of a current transformer, where the ratio of primary to secondary turns is:

$$n = \frac{N_s}{N_p}$$  \[16-1\]
Analysis of the Input Current Component

A better understanding of the current transformer behavior may be achieved by considering the applied input current to the primary winding, in terms of various components. Only the ampere-turn component, $I_nN_p$, drives the magnetic flux around the core. The ampere-turn, $I_mN_p$, provides the core loss. The secondary ampere-turns, $I_SN_p$, balance the remainder of the primary ampere-turns.

The exciting current, $I_m$, in Figure 16-2, determines the maximum accuracy that can be achieved with a current transformer. Exciting current, $I_m$, may be defined as the portion of the primary current that satisfies the hysteresis and eddy current losses of the core. If the values of $L_c$ and $R_c$ in Figure 16-2, are too low because the permeability of the core material is low and the core loss is high, only a part of the current, $(I_p/n)$, will flow in the output load resistor, $R_o$. The relationship of the exciting current, $I_m$, to the load current, $I_o$, is shown in Figure 16-3.

The exciting current is equal to:

$$I_m = \frac{H(MPL)}{0.4\pi N}, \quad \text{amps} \quad [16-2]$$

where $H$ is the magnetizing force and material dependent, and MPL is the Magnetic Path Length.
The input current is comprised of two components: exciting current, \( I_m \), and the load current, \( I_o \).

\[
I_{in}^2 = I_m^2 + I_o^2, \quad \text{[amps]} \quad [16-3]
\]

Then:

\[
I_m^2 = I_{in}^2 - I_o^2, \quad \text{[amps]} \quad [16-4]
\]

\[
I_m = I_{in} \sqrt{1 - \left( \frac{I_o}{I_{in}} \right)^2}, \quad \text{[amps]} \quad [16-5]
\]

The above equation has shown graphically in Figure 16-4, that the higher the exciting current, \( I_m \) or core loss, the larger the error. The magnetizing impedance, \( R_m \), determines accuracy, because it shunts part of the input current, \( I_m \), away from the primary and thus, produces an error, as shown in Figure 16-4. Core material with the lowest value of \( H \) achieves the highest accuracy.

![Figure 16-4. Input Current \( I_{in} \) Phase Relationship Diagram.](image)

**Unique to a Current Transformer**

The current transformer function is different than that of a voltage transformer. A current transformer operates with a set primary current and will try to output a constant current to the load, independent of the load. The current transformer will operate into either a short circuit or a resistive load until the voltage induced is enough to saturate the core or cause voltage breakdown. For this reason a current transformer should never operate into an open circuit, as a voltage transformer should never operate into a short circuit.

The primary current of a current transformer is not dependent of the secondary load current. The current is really injected into the primary by an external load current, \( I_{in} \). If the load current, \( I_o \), on the current transformer is removed from the secondary winding, while the external load current, \( I_{in} \), is still applied, the flux in the core will rise to a high level, because there is not an opposing current in the secondary winding to prevent this. A very high voltage will appear across the secondary. A current transformer, like any other transformer, must satisfy the amp-turn equation:

\[
\frac{I_p}{I_s} = \frac{N_t}{N_p} \quad [16-6]
\]
The secondary load, \( R_o \), secondary winding resistance, \( R_s \), and secondary load current, \( I_o \), determine the induced voltage of the current transformer.

\[
V_s = I_o (R_s + R_o), \quad \text{[volts]} \quad [16-7]
\]

If the secondary is designed for dc, then the diode drop must be taken into account.

\[
V_s = I_o (R_s + R_o) + V_d, \quad \text{[volts]} \quad [16-8]
\]

Simple form:

\[
V_s = V_o + V_d, \quad \text{[volts]} \quad [16-9]
\]

The current ratio will set the turns ratio. The secondary, \( R_o \) load will determine the secondary voltage, \( V_s \). The engineer would use Equation 16-10, to select the required core cross-section, \( A_c \). It is now up to the engineer to pick a core material that would provide the highest permeability at the operating flux density, \( B_{ac} \).

\[
A_c = \frac{I_{im} (R_s + R_o) (10^4)}{K_f B_{ac} f N_s}, \quad \text{[cm}^2\text{]} \quad [16-10]
\]

The design requirements would dictate choosing a core material and operating flux density, \( B_{ac} \), that would result in values of, \( L_c \) and \( R_e \), as shown in Figure 16-2, values which would be large enough to reduce the current flowing in these elements to satisfy the ratio and phase requirements.

The inductance is calculated from the equation:

\[
L_c = \frac{0.4\pi N_s^2 A_c \Delta \mu (10^{-8})}{MPL}, \quad \text{[henrys]} \quad [16-11]
\]

\( R_e \) is the equivalent core loss, (shunt), resistance. The current is, in phase, with the voltage.

\[
R_e = \frac{V_s}{I_{im} n}, \quad \text{[ohms]} \quad [16-12]
\]

Where:

\[
\frac{R_s}{n^2} \square R_s + R_o \quad [16-13]
\]

And:

\[
\frac{2\pi f L_c}{n^2} \square R_s + R_o \quad [16-14]
\]

Then:

\[
I_p = nI_s \quad [16-15]
\]
Or:

\[ I_p N_p = I_s N_s \quad [16-16] \]

Except for relatively low-accuracy industrial types, current transformers are wound on toroidal cores, which virtually eliminate errors due to leakage inductance. Some errors may be compensated for by adjusting the number of secondary turns.

**Current Transformer Circuit Applications**

Typical current transformer applications are shown in Figures 16-5 through 16-8.

**Figure 16-5.** Current Transformer, T2, used to Monitor, Q1 and Q2, Drain Current.

**Figure 16-6.** Current Transformer, T1, used to Monitor Line Current.
Figure 16-7. Current Transformer, T1, is used as a Level Detector.

Figure 16-8. Current Transformer, T1, is used for regenerative drive.
Current Transformer Design Example

The following information is the design specification for a current transformer, as shown in Figure 16-9.

1. Primary ................................................................. 1 turn
2. Input current, $I_{in}$ .................................................. 0–5 amps
3. Output voltage, $V_o$ .................................................. 0–5 volts
4. Output load resistance, $R_o$ ...................................... 500 ohms
5. Operating frequency, $f$ (square wave) ....................... 2500 hertz
6. Operating flux density, $B_{ac}$ ...................................... 0.2 tesla
7. Core loss less than ............................................... 3% (error)
8. Diode drop, $V_d$ .................................................... 1 volt
9. Magnetic material ................................................. Supermalloy 2 mil
10. Waveform factor, $K_f$ ............................................. 4.0

Step 1. Calculate the secondary current, $I_s$.

$$I_s = \frac{V_o}{R_o}, \text{ [amps]}$$

$$I_s = \frac{5.0}{500}, \text{ [amps]}$$

$$I_s = 0.01, \text{ [amps]}$$

Step 2. Calculate the secondary turns, $N_s$.

$$N_s = \frac{I_p N_p}{I_s}, \text{ [turns]}$$

$$N_s = \frac{(5.0)(1.0)}{0.01}, \text{ [turns]}$$

$$N_s = 500, \text{ [turns]}$$
Step 3. Calculate the secondary voltage, $V_s$.

\[ V_s = V_o + 2V_d, \quad \text{[volts]} \]
\[ V_s = 5.0 + 2(1.0), \quad \text{[volts]} \]
\[ V_s = 7.0, \quad \text{[volts]} \]

Step 4. Calculate the required core iron cross-section, $A_c$, using Faraday's Equation.

\[
A_c = \frac{V_s \left(10^4\right)}{(K_f)B_{at}fN_s}, \quad \text{[cm}^2]\]
\[
A_c = \frac{(7.0)(10^4)}{(4.0)(0.2)(2500)(500)}, \quad \text{[cm}^2]\]
\[ A_c = 0.070, \quad \text{[cm}^2]\]

Step 5. Select a 2mil tape, toroidal core from Chapter 3 with an iron cross-section, $A_c$, closest to the value calculated.

<table>
<thead>
<tr>
<th>Core number</th>
<th>52000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer</td>
<td>Magnetics</td>
</tr>
<tr>
<td>Magnetic material</td>
<td>2mil Silicon</td>
</tr>
<tr>
<td>Magnetic path length, MPL</td>
<td>4.99 cm</td>
</tr>
<tr>
<td>Core weight, $W_{fe}$</td>
<td>3.3 grams</td>
</tr>
<tr>
<td>Copper weight, $W_{cu}$</td>
<td>8.1 grams</td>
</tr>
<tr>
<td>Mean length turn, MLT</td>
<td>2.7 cm</td>
</tr>
<tr>
<td>Iron area, $A_c$</td>
<td>0.086 cm$^2$</td>
</tr>
<tr>
<td>Window area, $W_a$</td>
<td>0.851 cm$^2$</td>
</tr>
<tr>
<td>Area product, $A_p$</td>
<td>0.0732 cm$^4$</td>
</tr>
<tr>
<td>Core geometry, $K_c$</td>
<td>0.000938 cm$^5$</td>
</tr>
<tr>
<td>Surface area, $A_s$</td>
<td>20.6 cm$^2$</td>
</tr>
</tbody>
</table>

Step 6. Calculate the effective window area, $W_{a(eff)}$. A typical value for, $S_3$, is 0.75, as shown in Chapter 4.

\[
W_{a(eff)} = W_aS_3, \quad \text{[cm}^2]\]
\[
W_{a(eff)} = (0.851)(0.75), \quad \text{[cm}^2]\]
\[ W_{a(eff)} = 0.638, \quad \text{[cm}^2]\]
Step 7. Calculate the secondary window area, $W_{a(\text{sec})}$:

$$ W_{a(\text{sec})} = \frac{W_{a(\text{eff})}}{2}, \quad [\text{cm}^2] $$

$$ W_{a(\text{sec})} = \frac{0.638}{2}, \quad [\text{cm}^2] $$

$$ W_{a(\text{sec})} = 0.319, \quad [\text{cm}^2] $$

Step 8. Calculate the wire area, $A_w$, with insulation, using a fill factor, $S_2$ of 0.6.

$$ A_w = \frac{W_{a(\text{sec})} S_2}{N_t}, \quad [\text{cm}^2] $$

$$ A_w = \frac{(0.319)(0.6)}{(500)}, \quad [\text{cm}^2] $$

$$ A_w = 0.000383, \quad [\text{cm}^2] $$

Step 9. Select a wire area, $A_w$, with insulation from Wire Table in Chapter 4 for an equivalent AWG wire size. The rule is that when the calculated wire area does not fall within 10% of those listed, in Wire Table, then, the next smaller size should be selected.

AWG No. 33

$$ A_w = 0.0003662, \quad [\text{cm}^2] $$

Step 10. Calculate the secondary winding resistance, $R_s$, using the Wire Table in Chapter 4, for $\mu\Omega$/cm; and Step 5 for the MLT.

$$ R_s = \text{MLT} \left( N_t \right) \left( \frac{\mu\Omega}{\text{cm}} \right) \left( 10^{-6} \right), \quad [\text{ohms}] $$

$$ R_s = (2.7)(500)(6748)(10^{-6}), \quad [\text{ohms}] $$

$$ R_s = 9.11, \quad [\text{ohms}] $$

Step 11. Calculate the secondary output power, $P_o$.

$$ P_o = I_s (V_o + 2V_d), \quad [\text{watts}] $$

$$ P_o = (0.01)(5.0 + 2(1.0)), \quad [\text{watts}] $$

$$ P_o = 0.070, \quad [\text{watts}] $$
Step 12. Calculate the acceptable core loss, \( P_{fe} \).

\[
P_{fe} = P_o \left( \frac{\text{core loss} \%}{100} \right), \quad \text{[watts]}
\]

\[
P_{fe} = (0.07) \left( \frac{3}{100} \right), \quad \text{[watts]}
\]

\[
P_{fe} = 0.0021, \quad \text{[watts]}
\]

Step 13. Calculate the effective core weight, \( W_{te(\text{eff})} \). Select the core weight correction factor, \( K_w \), in Chapter 2, for Supermalloy.

\[
W_{te(\text{eff})} = W_{te} K_w, \quad \text{[grams]}
\]

\[
W_{te(\text{eff})} = (3.3)(1.148), \quad \text{[grams]}
\]

\[
W_{te(\text{eff})} = 3.79, \quad \text{[grams]}
\]

Step 14. Calculate the allowable core loss, \( P_{fe} \), in milliwatts per gram, \( \text{mW/g} \).

\[
\text{mW/g} = \frac{P_{fe}}{W_{te}}, \quad \text{[milliwatts per gram]}
\]

\[
\text{mW/g} = \frac{(0.0021)}{3.79}(10^4), \quad \text{[milliwatts per gram]}
\]

\[
\text{mW/g} = 0.554, \quad \text{[milliwatts per gram]}
\]

Step 15. Calculate the new flux density using the new core iron, cross-section, \( A_c \).

\[
B_{ac} = \frac{V_s (10^4)}{(K_f)(A_c f N_s)}, \quad \text{[tesla]}
\]

\[
B_{ac} = \frac{(7.0)(10^4)}{(4.0)(0.086)(2500)(500)}, \quad \text{[tesla]}
\]

\[
B_{ac} = 0.162, \quad \text{[tesla]}
\]

Step 16. Calculate the core loss, \( P_{fe} \), in milliwatts per gram, \( \text{mW/g} \).

\[
\text{mW/g} = 0.000179 f^{(4.48)} (B_{ac})^{(2.15)}, \quad \text{[milliwatts per gram]}
\]

\[
\text{mW/g} = 0.000179(2500)^{4.48}(0.162)^{2.15}, \quad \text{[milliwatts per gram]}
\]

\[
\text{mW/g} = 0.382, \quad \text{[milliwatts per gram]}
\]
Step 17. Calculate the core loss, $P_{fe}$, in watts.

$$P_{fe} = W_{fe} \left( \frac{mW}{g} \right) \left( 10^{-3} \right), \text{ [watts]}$$

$$P_{fe} = 3.79(0.382)(10^{-3}), \text{ [watts]}$$

$$P_{fe} = 0.00145, \text{ [watts]}$$

Step 18. Calculate the induced core error in, %.

Core loss induced error = \frac{P_{fe}}{P_{o}}(100), \text{ [%]}

Core loss induced error = \frac{0.00145}{0.7}(100), \text{ [%]}

Core loss induced error = 2.07, \text{ [%]}

**Design Performance**

A current transformer was built and the data recorded in Table 16-1. It was plotted in Figure 16-10, with an error of 3.4 %. The secondary winding resistance was 6.5 ohms.

**Table 16-1**

<table>
<thead>
<tr>
<th>$I_m$ amps</th>
<th>$I_o$ volts</th>
<th>$I_m$ amps</th>
<th>$I_o$ volts</th>
<th>$I_m$ amps</th>
<th>$I_o$ volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250</td>
<td>0.227</td>
<td>1.441</td>
<td>1.377</td>
<td>3.625</td>
<td>3.488</td>
</tr>
<tr>
<td>0.500</td>
<td>0.480</td>
<td>2.010</td>
<td>1.929</td>
<td>3.942</td>
<td>3.791</td>
</tr>
<tr>
<td>0.746</td>
<td>0.722</td>
<td>2.400</td>
<td>2.310</td>
<td>4.500</td>
<td>4.339</td>
</tr>
<tr>
<td>1.008</td>
<td>0.978</td>
<td>2.693</td>
<td>2.593</td>
<td>5.014</td>
<td>4.831</td>
</tr>
<tr>
<td>1.262</td>
<td>1.219</td>
<td>3.312</td>
<td>3.181</td>
<td>5.806</td>
<td>5.606</td>
</tr>
</tbody>
</table>

**Figure 16-10.** Current Transformer, Input Current versus Output Voltage.