Introduction

Considerable difficulty is encountered in mastering the field of magnetics because of the use of so many different systems of units – the centimeter-gram-second (cgs) system, the meter-kilogram-second (mks) system, and the mixed English units system. Magnetics can be treated in a simple way by using the cgs system. There always seems to be one exception to every rule and that is permeability.

Magnetic Properties in Free Space

A long wire with a dc current, I, flowing through it, produces a circulatory magnetizing force, H, and a magnetic field, B, around the conductor, as shown in Figure 1-1, where the relationship is:

\[
B = \mu_0 H, \quad [\text{gauss}]
\]

\[
H = \frac{I}{2\pi r}, \quad [\text{oersteds}]
\]

\[
B_m = \frac{\phi}{\text{cm}^2}, \quad [\text{gauss}]
\]

**Figure 1-1.** A Magnetic Field Generated by a Current Carrying Conductor.

The direction of the line of flux around a straight conductor may be determined by using the “right hand rule” as follows: When the conductor is grasped with the right hand, so that the thumb points in the direction of the current flow, the fingers point in the direction of the magnetic lines of force. This is based on so-called conventional current flow, not the electron flow.

When a current is passed through the wire in one direction, as shown in Figure 1-2(a), the needle in the compass will point in one direction. When the current in the wire is reversed, as in Figure 1-2(b), the needle will also reverse direction. This shows that the magnetic field has polarity and that, when the current I, is reversed, the magnetizing force, H, will follow the current reversals.
Intensifying the Magnetic Field

When a current passes through a wire, a magnetic field is set up around the wire. If the conductors, as shown in Figure 1-3, carrying current in the same direction are separated by a relatively large distance, the magnetic fields generated will not influence each other. If the same two conductors are placed close to each other, as shown in Figure 1-4, the magnetic fields add, and the field intensity doubles.

\[ \gamma = \frac{B^2}{8\pi \mu} \quad \text{[energy density]} \quad [1-1] \]

If the wire is wound on a dowel, its magnetic field is greatly intensified. The coil, in fact, exhibits a magnetic field exactly like that of a bar magnet, as shown in Figure 1-5. Like the bar magnet, the coil has a north pole and a neutral center region. Moreover, the polarity can be reversed by reversing the current, I, through the coil. Again, this demonstrates the dependence of the magnetic field on the current direction.
The magnetic circuit is the space in which the flux travels around the coil. The magnitude of the flux is determined by the product of the current, I, and the number of turns, N, in the coil. The force, NI, required to create the flux is magnetomotive force (mmf). The relationship between flux density, B, and magnetizing force, H, for an air-core coil is shown in Figure 1-6. The ratio of B to H is called the permeability, \( \mu \), and for this air-core coil the ratio is unity in the cgs system, where it is expressed in units of gauss per oersteds, \((\text{gauss/oersteds})\).

\[
\mu = \frac{1}{B} = \mu_o H \quad [1-2]
\]

If the battery, in Figure 1-5, were replaced with an ac source, as shown in Figure 1-7, the relationship between B and H would have the characteristics shown in Figure 1-8. The linearity of the relationship between B and H represents the main advantage of air-core coils. Since the relationship is linear, increasing H increases B, and therefore the flux in the coil, and, in this way, very large fields can be produced with large currents. There is obviously a practical limit to this, which depends on the maximum allowable current in the conductor and the resulting rise.
Figure 1-5. Air-Core Coil with dc excitation

Figure 1-6. Relationship Between B and H with dc Excitation.

Figure 1-7. Air-Core Coil Driven from an ac Source.
Fields of the order of 0.1 tesla are feasible for a 40°C temperature rise above room ambient temperature. With super cooled coils, fields of 10 tesla have been obtained.

![Graph showing the relationship between B and H with ac Excitation.](image)

**Figure 1-8.** Relationship Between B and H with ac Excitation.

**Simple Transformer**

A transformer in its simplest form is shown in Figure 1-9. This transformer has two air coils that share a common flux. The flux diverges from the ends of the primary coil in all directions. It is not concentrated or confined. The primary is connected to the source and carries the current that establishes a magnetic field. The other coil is open-circuited. Notice that the flux lines are not common to both coils. The difference between the two is the leakage flux; that is, leakage flux is the portion of the flux that does not link both coils.

![Diagram of the Simplest Type of Transformer.](image)

**Figure 1-9.** The Simplest Type of Transformer.
Magnetic Core

Most materials are poor conductors of magnetic flux; they have low permeability. A vacuum has a permeability of 1.0, and nonmagnetic materials, such as air, paper, and copper have permeabilities of the same order. There are a few materials, such as iron, nickel, cobalt, and their alloys that have high permeability, sometimes ranging into the hundreds of thousands. To achieve an improvement over the air-coil, as shown in Figure 1-10, a magnetic core can be introduced, as shown in Figure 1-11. In addition to its high permeability, the advantages of the magnetic core over the air-core are that the magnetic path length (MPL) is well-defined, and the flux is essentially confined to the core, except in the immediate vicinity of the winding. There is a limit as to how much magnetic flux can be generated in a magnetic material before the magnetic core goes into saturation, and the coil reverts back to an air-core, as shown in Figure 1-12.

Figure 1-10. Air-Core Coil Emitting Magnetic Flux when Excited.

Figure 1-11. Introduction of a Magnetic Core.
Fundamental Characteristics of a Magnetic Core

The effect of exciting a completely demagnetized, ferromagnetic material, with an external magnetizing force, $H$, and increasing it slowly, from zero, is shown in Figure 1-13, where the resulting flux density is plotted as a function of the magnetizing force, $H$. Note that, at first, the flux density increases very slowly up to point $A$, then, increases very rapidly up to point $B$, and then, almost stops increasing. Point $B$ is called the knee of the curve. At point $C$, the magnetic core material has saturated. From this point on, the slope of the curve is:

$$\frac{B}{H} = 1, \text{ [gauss/oersteds]} \quad [1-3]$$

The coil is now behaving as if it had an air-core. When the magnetic core is in hard saturation, the coil has the same permeability as air, or unity. Following the magnetization curve in Figure 1-14, Figures 1-15 through Figures 1-16 show how the flux in the core is generated from the inside of the core to the outside until the core saturates.
Figure 1-14. Magnetic Core with Zero Excitation.

Figure 1-15. Magnetic Core with Low Excitation.

Figure 1-16. Magnetic Core with High Excitation.
**Hysteresis Loop (B-H Loop)**

An engineer can take a good look at the hysteresis loop and get a first order evaluation of the magnetic material. When the magnetic material is taken through a complete cycle of magnetization and demagnetization, the results are as shown in Figure 1-17. It starts with a neutral magnetic material, traversing the B-H loop at the origin X. As H is increased, the flux density B increases along the dashed line to the saturation point, B_s. When H is now decreased and B is plotted, B-H loop transverses a path to B_r, where H is zero and the core is still magnetized. The flux at this point is called remanent flux, and has a flux density, B_r.

The magnetizing force, H, is now reversed in polarity to give a negative value. The magnetizing force required to reduce the flux B_r to zero is called the coercive force, H_c. When the core is forced into saturation, the retentivity, B_r, is the remaining flux after saturation, and coercivity, H_c, is the magnetizing force required to reset to zero. Along the initial magnetization curve at point X, the dashed line, in Figure 1-17, B increases from the origin nonlinearly with H, until the material saturates. In practice, the magnetization of a core in an excited transformer never follows this curve, because the core is never in the totally demagnetized state, when the magnetizing force is first applied.

The hysteresis loop represents energy lost in the core. The best way to display the hysteresis loop is to use a dc current, because the intensity of the magnetizing force must be so slowly changed that no eddy currents are generated in the material. Only under this condition is the area inside the closed B-H loop indicative of the hysteresis. The enclosed area is a measure of energy lost in the core material during that cycle. In ac applications, this process is repeated continuously and the total hysteresis loss is dependent upon the frequency.

![Hysteresis Loop Diagram](image)

**Figure 1-17.** Typical Hysteresis Loop.
Permeability

In magnetics, permeability is the ability of a material to conduct flux. The magnitude of the permeability at a given induction is the measure of the ease with which a core material can be magnetized to that induction. It is defined as the ratio of the flux density, B, to the magnetizing force, H. Manufacturers specify permeability in units of gauss per oersteds.

\[
\text{Permeability} = \frac{B}{H}, \quad \left[\text{gauss per oersteds}\right] \quad [1-4]
\]

The absolute permeability, \(\mu_0\) in cgs units is unity (gauss per oersteds) in a vacuum.

\[
cgs: \quad \mu_0 = 1, \quad \left[\text{gauss per oersteds}\right] = \left[\text{tesla per oersteds}\right] \quad \left(10^4\right) \quad [1-5]
\]

\[
mks: \quad \mu_0 = 0.4\pi \left(10^{-8}\right), \quad \left[\text{henry per meter}\right]
\]

When B is plotted against H, as in Figure 1-18, the resulting curve is called the magnetization curve. These curves are idealized. The magnetic material is totally demagnetized and is then subjected to gradually increasing magnetizing force, while the flux density is plotted. The slope of this curve, at any given point gives the permeability at that point. Permeability can be plotted against a typical B-H curve, as shown in Figure 1-19. Permeability is not constant; therefore, its value can be stated only at a given value of B or H.

There are many different kinds of permeability, and each is designated by a different subscript on the symbol \(\mu\).

- \(\mu_0\): Absolute permeability, defined as the permeability in a vacuum.
- \(\mu_i\): Initial permeability is the slope of the initial magnetization curve at the origin. It is measured at very small induction, as shown in Figure 1-20.
- \(\mu_A\): Incremental permeability is the slope of the magnetization curve for finite values of peak-to-peak flux density with superimposed dc magnetization as shown in Figure 1-21.
- \(\mu_e\): Effective permeability. If a magnetic circuit is not homogeneous (i.e., contains an air gap), the effective permeability is the permeability of hypothetical homogeneous (ungapped) structure of the same shape, dimensions, and reluctance that would give the inductance equivalent to the gapped structure.
- \(\mu_r\): Relative permeability is the permeability of a material relative to that of free space.
- \(\mu_n\): Normal permeability is the ratio of B/H at any point of the curve as shown in Figure 1-22.
- \(\mu_{\max}\): Maximum permeability is the slope of a straight line drawn from the origin tangent to the curve at its knee as shown in Figure 1-23.
- \(\mu_p\): Pulse permeability is the ratio of peak B to peak H for unipolar excitation.
- \(\mu_m\): Material permeability is the slope of the magnetization curve measure at less than 50 gauss as shown in Figure 1-24.
Figure 1-18. Magnetizing Curve.

Figure 1-19. Variation of Permeability $\mu$ along the Magnetizing Curve.

Figure 1-20. Initial Permeability.
Figure 1-21. Incremental Permeability.

Figure 1-22. Normal Permeability.

Figure 1-23. Maximum Permeability.
Magnetomotive Force (mmf) and Magnetizing Force (H)

There are two force functions commonly encountered in magnetics: magnetomotive force, mmf, and magnetizing force, H. Magnetomotive force should not be confused with magnetizing force; the two are related as cause and effect. Magnetomotive force is given by the equation:

$$\text{mmf} = 0.4\pi NI, \quad \text{[gilberts]} \quad [1-6]$$

Where, N is the number of turns and I is the current in amperes. Whereas mmf is the force, H is a force field, or force per unit length:

$$H = \frac{\text{mmf}}{\text{MPL}}, \quad \left[\frac{\text{gilberts}}{\text{cm}} = \text{oersteds}\right] \quad [1-7]$$

Substituting,

$$H = \frac{0.4\pi NI}{\text{MPL}}, \quad \left[\text{oersteds}\right] \quad [1-8]$$

Where, MPL = magnetic path length in cm.

If the flux is divided by the core area, $A_c$, we get flux density, B, in lines per unit area:

$$B = \frac{\phi}{A_c}, \quad \left[\text{gauss}\right] \quad [1-9]$$

The flux density, B, in a magnetic medium, due to the existence of a magnetizing force H, depends on the permeability of the medium and the intensity of the magnetic field:

$$B = \mu H, \quad \left[\text{gauss}\right] \quad [1-10]$$
The peak, magnetizing current, $I_{nl}$, for a wound core can be calculated from the following equation:

$$ I_{nl} = \frac{H_0 (MPL)}{0.4 \pi N}, \text{ [amps]} \quad [1-11] $$

Where $H_0$ is the field intensity at the peak operating point. To determine the magnetizing force, $H_0$, use the manufacturer's core loss curves at the appropriate frequency and operating flux density, $B_0$, as shown in Figure 1-25.

![B-H Loops Operating at Various Frequencies](image)

**Figure 1-25.** Typical B-H Loops Operating at Various Frequencies.

**Reluctance**

The flux produced in a given material by magnetomotive force (mmf) depends on the material's resistance to flux, which is called reluctance, $R_m$. The reluctance of a core depends on the composition of the material and its physical dimension and is similar in concept to electrical resistance. The relationship between mmf, flux, and magnetic reluctance is analogous to the relationship between emf, current, and resistance, as shown in Figure 1-26.

$$ emf \ (E) = IR = \text{Current x Resistance} $$
$$ mmf \ (F_m) = \Phi R_m = \text{Flux x Reluctance} \quad [1-12] $$

A poor conductor of flux has a high magnetic resistance, $R_m$. The greater the reluctance, the higher the magnetomotive force that is required to obtain a given magnetic field.
The electrical resistance of a conductor is related to its length $l$, cross-sectional area $A_w$, and specific resistance $\rho$, which is the resistance per unit length. To find the resistance of a copper wire of any size or length, we merely multiply the resistivity by the length, and divide by the cross-sectional area:

$$ R = \frac{\rho l}{A_w} \text{ [ohms]} \quad [1-13] $$

In the case of magnetics, $1/\mu$ is analogous to $\rho$ and is called reluctivity. The reluctance $R_m$ of a magnetic circuit is given by:

$$ R_m = \frac{MPL}{\mu_r \mu_0 A_c} \quad [1-14] $$

Where $MPL$, is the magnetic path length, cm.
$A_c$ is the cross-section of the core, cm$^2$.
$\mu_r$ is the permeability of the magnetic material.
$\mu_0$ is the permeability of air.

A typical magnetic core is shown in Figure 1-27 illustrating the magnetic path length $MPL$ and the cross-sectional area, $A_c$, of a C core.
Air Gap

A high permeability material is one that has a low reluctance for a given magnetic path length (MPL) and iron cross-section, $A_c$. If an air gap is included in a magnetic circuit as shown in Figure 1-28, which is otherwise composed of low reluctivity material like iron, almost all of the reluctance in the circuit will be at the gap, because the reluctivity of air is much greater than that of a magnetic material. For all practical purposes, controlling the size of the air gap controls the reluctance.

![Figure 1-28. A Typical Magnetic Core with an Air Gap.](image)

An example can best show this procedure. The total reluctance of the core is the sum of the iron reluctance and the air gap reluctance, in the same way that two series resistors are added in an electrical circuit. The equation for calculating the air gap reluctance, $R_g$, is basically the same as the equation for calculating the reluctance of the magnetic material, $R_m$. The difference is that the permeability of air is 1 and the gap length, $l_g$, is used in place of the magnetic path length (MPL). The equation is as follows:

$$ R_g = \left( \frac{1}{\mu_0} \right) \frac{l_g}{A_c} \quad [1-15] $$

But, since $\mu_0 = 1$, the equation simplifies to:

$$ R_g = \frac{l_g}{A_c} \quad [1-16] $$

Where:

- $l_g$ is the gap length, cm.
- $A_c$ is the cross-section of the core, cm$^2$.
- $\mu_0$ is the permeability of air.

The total reluctance, $R_{mt}$, for the core shown in Figure 1-28 is therefore:
\[ R_{mr} = R_m + R_g \]
\[ R_{mr} = \frac{MPL}{\mu_r \mu_e A_e} + \frac{l_g}{\mu_e A_e} \quad [1-17] \]

Where \( \mu_r \) is the relative permeability, which is used exclusively with magnetic materials.

\[ \mu_r = \frac{B}{\mu_o H} = \left[ \frac{\text{gauss}}{\text{oersteds}} \right] \quad [1-18] \]

The magnetic material permeability, \( \mu_m \), is given by:

\[ \mu_m = \mu_r \mu_o \quad [1-19] \]

The reluctance of the gap is higher than that of the iron even when the gap is small. The reason is because the magnetic material has a relatively high permeability, as shown in Table 1-1. So the total reluctance of the circuit depends more on the gap than on the iron.

**Table 1-1. Material Permeability**

<table>
<thead>
<tr>
<th>Material Name</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron Alloys</td>
<td>0.8K to 25K</td>
</tr>
<tr>
<td>Ferrites</td>
<td>0.8K to 20K</td>
</tr>
<tr>
<td>Amorphous</td>
<td>0.8K to 80K</td>
</tr>
</tbody>
</table>

After the total reluctance, \( R_o \), has been calculated, the effective permeability, \( \mu_e \), can be calculated.

\[ R_{mr} = \frac{l_g}{\mu_e A_e} \]

\[ l_g = l_g + MPL \quad [1-20] \]

Where \( l_g \) is the total path length and \( \mu_e \) is the effective permeability.

\[ R_{mr} = \frac{l_g}{\mu_e A_e} = \frac{l_g + MPL}{\mu_o \mu_e \mu_r A_e} \quad [1-21] \]

Simplifying yields:

\[ \frac{l_g}{\mu_e} = \frac{l_g + MPL}{\mu_o \mu_r} \quad [1-22] \]

Then:

\[ \mu_e = \frac{l_g}{l_g + MPL} \]

\[ \mu_e = \frac{l_g + MPL}{l_g + MPL} \quad [1-23] \]

\[ \mu_e = \frac{l_g + MPL}{\mu_o \mu_r} \]

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If $I_s << MPL$, multiply both sides of the equation by $(\mu_s \mu_0 MPL)/ (\mu_s \mu_0 MPL)$.

$$\mu_e = \frac{\mu_s \mu_0}{1 + \mu_s \left(\frac{l_s}{MPL}\right)} \quad [1-24]$$

The classic equation is:

$$\mu_e = \frac{\mu_m}{1 + \mu_m \left(\frac{l_s}{MPL}\right)} \quad [1-25]$$

Introducing an air gap, $l_s$, to the core cannot correct for the dc flux, but can sustain the dc flux. As the gap is increased, so is the reluctance. For a given magnetomotive force, the flux density is controlled by the gap.

**Controlling the dc Flux with an Air Gap**

There are two similar equations used to calculate the dc flux. The first equation is used with powder cores. Powder cores are manufactured from very fine particles of magnetic materials. This powder is coated with an inert insulation to minimize eddy currents losses and to introduce a distributed air gap into the core structure.

$$\mu_e = \mu_s$$

$$B_{dc} = (\mu_s) \left(\frac{0.4\pi N I}{MPL}\right), \quad [\text{gauss}] \quad [1-26]$$

$$\mu_e = \frac{\mu_m}{1 + \mu_m \left(\frac{l_s}{MPL}\right)}$$

The second equation is used, when the design calls for a gap to be placed in series with the magnetic path length (MPL), such as a ferrite cut core, a C core, or butt stacked laminations.

$$\mu_e = \mu_s$$

$$B_{dc} = (\mu_s) \left(\frac{0.4\pi N I}{MPL}\right), \quad [\text{gauss}] \quad [1-27]$$

Substitute $(MPL\mu_m)/(MPL\mu_m)$ for $1$:

$$\mu_e = \frac{\mu_m}{1 + \mu_m \left(\frac{l_s}{MPL}\right)} = \frac{\mu_m}{MPL\mu_m + \mu_m \left(\frac{l_s}{MPL}\right)} \quad [1-28]$$
Then, simplify:

$$\mu_r = \frac{\text{MPL}}{\text{MPL} + I_x}$$  \hspace{1cm} [1-29]

$$B_{dc} = \left( \frac{\text{MPL}}{\text{MPL} + I_x} \right) \left( \frac{0.4\pi \, N \, I}{\text{MPL}} \right), \ [\text{gauss}] \quad [1-30]$$

Then, simplify:

$$B_{dc} = \frac{0.4\pi \, N \, I}{I_x + \frac{\text{MPL}}{\mu_m}}, \ [\text{gauss}] \quad [1-31]$$

**Types of Air Gaps**

Basically, there are two types of gaps used in the design of magnetic components: bulk and distributed. Bulk gaps are maintained with materials, such as paper, Mylar, or even glass. The gapping materials are designed to be inserted in series with the magnetic path to increase the reluctance, R, as shown in Figure 1-29.

![Diagram](image-url)

**Figure 1-29.** Placement of the Gapping Materials.

Placement of the gapping material is critical in keeping the core structurally balanced. If the gap is not proportioned in each leg, then the core will become unbalanced and create even more than the required gap. There are designs where it is important to place the gap in an area to minimize the noise that is caused by the fringing flux at the gap. The gap placement for different core configurations is shown in Figure 1-30. The standard gap placement is shown in Figure 1-30A, C, and D. The EE or EC cores shown in Figure 1-30B, are best-suited, when the gap has to be isolated within the magnetic assembly to minimize fringing flux noise. When the gap is used as shown in Figure 1-30A, C, and D, then, only half the thickness of the calculated gap dimension is used in each leg of the core.
Fringing flux has been around since time began for the power conversion engineer. Designing power conversion magnetics that produce a minimum of fringing flux has always been a problem. Engineers have learned to design around fringing flux, and minimize its effects. It seems that when engineers do have a problem, it is usually at the time when the design is finished and ready to go. It is then that the engineer will observe something that was not recognized before. This happens during the final test when the unit becomes unstable, the inductor current is nonlinear, or the engineer just located a hot spot during testing. Fringing flux can cause a multitude of problems. Fringing flux can reduce the overall efficiency of the converter, by generating eddy currents that cause localized heating in the windings and/or the brackets. When designing inductors, fringing flux must be taken into consideration. If the fringing flux is not handled correctly, there will be premature core saturation. More and more magnetic components are now designed to operate in the sub-megahertz region. High frequency has really brought out the fringing flux and its parasitic eddy currents. Operating at high frequency has made the engineer very much aware of what fringing flux can do to hamper a design.
Material Permeability, \( (\mu_m) \)

The B-H loops that are normally seen in the manufacturers’ catalogs are usually taken from a toroidal sample of the magnetic material. The toroidal core, without a gap, is the ideal shape to view the B-H loop of a given material. The material permeability, \( \mu_m \), will be seen at its highest in the toroidal shape, as shown in Figure 1-31.

\[ \text{Figure 1-31. The Shearing of an Idealized B-H Loop Due to an Air Gap.} \]

A small amount of air gap, less than 25 microns, has a powerful effect by shearing over the B-H loop. This shearing over of the B-H loop reduces the permeability. High permeability ferrites that are cut, like E cores, have only about 80 percent of the permeability, than that of a toroid of the same material. This is because of the induced gap, even though the mating surfaces are highly polished. In general, magnetic materials with high-permeability, are sensitive to temperature, pressure, exciting voltage, and frequency. The inductance change is directly proportional to the permeability change. This change in inductance will have an effect on the exciting current. It is very easy to see, that inductors that are designed into an LC, tuned circuit, must have a stable permeability, \( \mu_e \).

\[ L = \frac{0.4\pi N^2 A_{g} \Delta \mu \left(10^{-3}\right)}{MPL}, \text{ [henrys]} \]  \[ \text{[1-32]} \]

Air Gaps

Air gaps are introduced into magnetic cores for a variety of reasons. In a transformer design a small air gap, \( l_g \), inserted into the magnetic path, will lower and stabilize the effective permeability, \( \mu_e \).
\[
\mu_e = \frac{\mu_n}{1 + \mu_n \left( \frac{l_x}{MPL} \right)} \quad [1-33]
\]

This will result in a tighter control of the permeability change with temperature, and exciting voltage. Inductor designs will normally require a large air gap, \( l_g \), to handle the dc flux.

\[
l_g = \frac{0.4 \pi N I_{dc} (10^{-4})}{B_{dc}}, \quad [\text{cm}] \quad [1-34]
\]

Whenever an air gap is inserted into the magnetic path, as shown in Figure 1-32, there is an induced, fringing flux at the gap.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fringing_flux.png}
\caption{Fringing Flux at the Gap.}
\end{figure}

The fringing flux effect is a function of gap dimension, the shape of the pole faces, and the shape, size, and location of the winding. Its net effect is to shorten the air gap. Fringing flux decreases the total reluctance of the magnetic path and, therefore, increases the inductance by a factor, \( F \), to a value greater than the one calculated.

**Fringing Flux, \( F \)**

Fringing flux is completely around the gap and re-enters the core in a direction of high loss, as shown in Figure 1-33. Accurate prediction of gap loss, \( P_g \), created by fringing flux is very difficult to calculate.
This area around the gap is very sensitive to metal objects, such as clamps, brackets and banding materials. The sensitivity is dependent on the intensity of the magnetomotive force, gap dimensions and the operating frequency. If a metal bracket or banding material is used to secure the core, and it passes over the gap, two things can happen: (1) If the material ferromagnetic is placed over the gap, or is in close proximity so it conducts the magnetic field, this is called “shorting the gap.” Shorting the gap is the same as reducing the gap dimension, thereby producing a higher inductance, than designed, and could drive the core into saturation. (2) If the material is metallic, (such as copper, or phosphor bronze), but not ferromagnetic, it will not short the gap or change the inductance. In both cases, if the fringing flux is strong enough, it will induce eddy currents that will cause localized heating. This is the same principle used in induction heating.

**Gapped, dc Inductor Design**

The fringing flux factor, F, has an impact on the basic inductor design equations. When the engineer starts a design, he or she must determine the maximum values for \( B_{dc} \) and for \( B_{ac} \), which will not produce magnetic saturation. The magnetic material that has been selected will dictate the saturation flux density. The basic equation for maximum flux density is:

\[
B_{\text{max}} = \frac{0.4\pi N \left( I_{dc} + \frac{\Delta I}{2} \right) \times 10^{-4}}{I_s + \frac{\text{MPL}}{\mu_m}}, \quad [\text{tesla}] \ [1-35]
\]

The inductance of an iron-core inductor, carrying dc and having an air gap, may be expressed as:

\[
L = \frac{0.4\pi N^2 A \times 10^{-8}}{I_s + \frac{\text{MPL}}{\mu_m}}, \quad [\text{henrys}] \ [1-36]
\]

The inductance is dependent on the effective length of the magnetic path, which is the sum of the air gap length, \( l_s \), and the ratio of the core magnetic path length to the material permeability, \( \frac{\text{MPL}}{\mu_m} \). The final determination of the air gap size requires consideration of the fringing flux effect which is a function of the

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gap dimension, the shape of the pole faces, and the shape, size, and location of the winding. The winding length, or the G dimension of the core, has a big influence on the fringing flux. See, Figure 1-34 and Equation 1-37.

The fringing flux decreases the total reluctance of the magnetic path length and, therefore, increases the inductance by a factor of \( F \) to a value greater than that calculated. The fringing flux factor is:

\[
F = \left(1 + \frac{l_g}{A_c} \ln \frac{2G}{l_g} \right) \quad [1-37]
\]

After the inductance has been calculated using Equation 1-36, the fringing flux factor has to be incorporated into Equation 1-36. Equation 1-36 can now be rewritten to include the fringing flux factor, as shown:

\[
L = \frac{0.4\pi N^2 l_e \left(10^{-8}\right)}{l_g + \frac{MPL}{\mu_m}} \quad [\text{henrys}] \quad [1-38]
\]

The fringing flux factor, \( F \), can now be included into Equation 1-35. This will check for premature, core saturation.

\[
B_{\text{max}} = F \frac{0.4\pi N \left(\frac{l_e + \Delta l}{2}\right) \left(10^{-4}\right)}{l_g + \frac{MPL}{\mu_m}} \quad [\text{tesla}] \quad [1-39]
\]

Now that the fringing flux factor, \( F \), is known and inserted into Equation 1-38. Equation 1-38 can be rewritten to solve for the required turns so that premature core saturation will not happen.

Figure 1-34. Dimensional, Call Out for C and E Cores.
Fringing Flux and Coil Proximity

As the air gap increases, the fringing flux will increase. Fringing flux will fringe out away from the gap by the distance of the gap. If a coil was wound tightly around the core and encompasses the gap, the flux generated around the magnet wire will force the fringing flux back into the core. The end result will not produce any fringing flux at all, as shown in Figure 1-35. As the coil distance moves away from the core, the fringing flux will increase until the coil distance from the core is equal to the gap dimension.

\[
N = \sqrt{\frac{L_1 (I_z + MPL)}{0.4\pi A_e F (10^{-6})}}, \quad \text{[turns]} \quad [1-40]
\]

Fringing Flux, Crowding

Flux will always take the path of highest permeability. This can best be seen in transformers with interleave laminations. The flux will traverse along the lamination until it meets its mating, I or E. At this point, the flux will jump to the adjacent lamination and bypass the mating point, as shown in Figure 1-36.
This phenomena can best be seen by observing the exciting current at low, medium and high flux levels, as shown in Figure 1-37. At low levels of excitation, the exciting current is almost square, due to the flux taking the high permeability path, by jumping to the adjacent lamination, as shown in Figure 1-36. As the excitation is increased, the adjoining lamination will start to saturate, and the exciting current will increase and become nonlinear. When the adjacent lamination approaches saturation, the permeability drops. It is then that the flux will go in a straight line and cross the minute air gap, as shown in Figure 1-36.

![Figure 1-37. Exciting Current, at Different Levels of Flux Density, B.](image)

**Fringing Flux and Powder Cores**

Designing high frequency converters, using low permeability powder cores, will usually require very few turns. Low perm power cores (less than 60), exhibit fringing flux. Powder cores with a distributed gap will have fringing flux that shorts the gap and gives the impression of a core with a higher permeability. Because of the fringing flux and a few turns, it is very important to wind uniformly and in a consistent manner. This winding is done to control the fringing flux and get inductance repeatability from one core to another, as shown in Figures 1-38 and 1-39.

![Figure 1-38. Comparing Toroidal, Winding Methods.](image)
Figure 1-39. Comparing EE Cores, Winding Methods.