

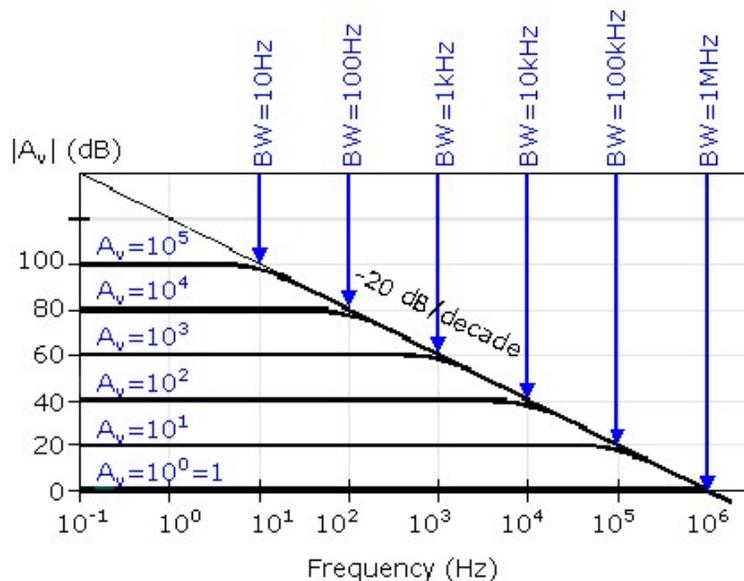
Section H7: Frequency Response of Op-Amp Circuits

In the previous sections, we have looked at the frequency response of single and multi-stage amplifier circuits using BJTs and FETs. We've seen how the high frequency cutoff is controlled by internal device capacitances, and the low frequency cutoff is determined by external capacitive components. We're going to finish up this section of our studies by looking at the frequency response of circuits that utilize our friendly IC, the operational amplifier.

Before we get started in earnest, recall the difference between open loop and closed loop operation:

- **open loop** operation means that there is no feedback between the output and the input. The gain of the op-amp device under this condition is defined as the *open loop gain* that we've been calling G_o .
- **closed loop** on the other hand, involves some sort of feedback mechanism between the output and the input. So far, we've only looked at negative feedback, but positive feedback is also used (but we'll get into that shortly). Gain under closed loop conditions is denoted by the letter "A" in our text with a subscript that defines the gain type – i.e., voltage gain is A_v , current gain is A_i and power gain is A_p .

With that brief (and cursory) review under our belts, a typical open-loop frequency response plot is shown in Figure 10.25 and is reproduced (slightly modified) to the right. In this figure, voltage gain is used as an example, with the vertical axis expressed in decibels (dB) and the corresponding closed loop gain noted in blue (i.e., $80\text{dB}=10^4 \text{ V/V}$, etc.). The horizontal axis in this figure is frequency in Hertz (cycles per second) using a logarithmic scale. For this first order system (indicated by the -20 dB/decade rolloff), the **bandwidth** is the upper break frequency (also called the corner frequency, half-power frequency, ...) is defined as the point at which we make the -3 dB correction from the straight line approximation.

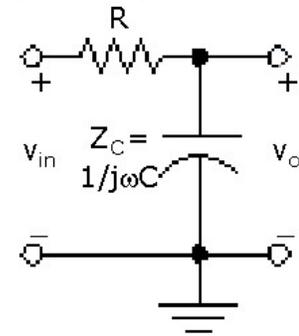


An important characteristic of this figure is that the **gain-bandwidth product (GBP)** is constant, and is 10^6 for this particular example as shown in Table 10.1 of your text and to the right. Note that this example is characteristic of a 741 with an open loop gain of 10^5 . Since a logarithmic plot never goes to zero, and the GBP must remain constant, the lowest frequency where this is true (10 Hz in this case) is considered the **zero frequency point**.

A_v	BW	GBP
10^5	10^1	10^6
10^4	10^2	10^6
10^3	10^3	10^6
10^2	10^4	10^6
10^1	10^5	10^6
10^0	10^6	10^6

The response of Figure 10.5 is typical of a general-purpose single roll-off op-amp such as the 741. Back in Section G5, it was mentioned that the 741 includes an internal compensation through an on-chip RC network. This form of fixed compensation is a technique that modifies the open-loop frequency response characteristic in order to improve performance and stability (hang on, we'll get to stability shortly).

Specifically with regard to an RC compensation network, such as is given in Figure 10.26 and to the right, the voltage transfer function may be written as



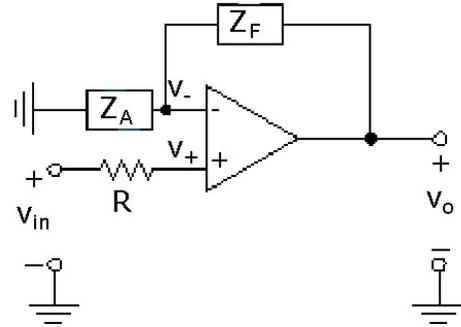
$$A_v(j\omega) = \frac{v_o(j\omega)}{v_{in}(j\omega)} = \frac{(1/j\omega C)}{(1/j\omega C) + R} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega\tau}, \quad (\text{Equation 10.79})$$

where the time constant, τ , is equal to RC . The RC network has a single pole at a radian frequency of $\omega = 1/\tau = 1/RC$. When the RC compensation network is used in an op-amp, the gain decreases at with a slope of -20 dB/decade for frequencies greater than ω . The 741 uses such a single pole fixed compensation network that is built into the device and is the cause of the frequency response illustrated in Figure 10.25. The family of curves in this figure may be approximated by

$$G(s) = \frac{G_o}{1 + s/\omega_o}, \quad (\text{Equation 10.80})$$

where G_o is the dc open-loop op-amp gain and ω_o is the op-amp corner frequency defined by the RC network. Note: In general, manufacturer's data sheets will show the amplitude and phase response of a particular op-amp under certain operating conditions.

A typical non-inverting op-amp configuration is shown in Figure 10.27 and is given to the right. Note that we have changed the purely resistive notation of circuit components to the more general impedance notation for the feedback and inverting input. Using our ideal approximation that the currents into the op-amp terminals are zero, or $i_+ = i_- = 0$, we can write the node equation at v_- as follows:



$$\frac{v_-}{Z_A} = \frac{v_o - v_-}{Z_F}; \text{ or } v_- = \frac{Z_A}{Z_A + Z_F} v_o, \quad (\text{Equation 10.82})$$

and we can see by inspection that $v_+ = v_{in}$.

Using the gain relationship developed last semester and substituting in the above expressions for v_+ and v_- , we have

$$v_o = G_o(v_+ - v_-) = G_o \left(v_{in} - \frac{Z_A}{Z_A + Z_F} v_o \right).$$

Finally, rearranging the above equation, we can get an expression for the output voltage, v_o

$$v_o = \frac{G_o v_{in}}{1 + \frac{G_o Z_A}{Z_A + Z_F}}. \quad (\text{Equation 10.83})$$

If we define a term (γ) to be

$$\gamma = \frac{Z_A}{Z_A + Z_F}, \quad (\text{Equation 10.84})$$

the closed loop gain, v_o/v_{in} , becomes (found by rearranging Equation 10.83 and using Equation 10.84)

$$A_v = \frac{G_o}{1 + G_o \gamma}, \quad (\text{Equation 10.85})$$

which approaches $1/\gamma$ for low frequencies if G_o is very large.

If the frequency dependent expression of Equation 10.80 for $G(s)$ is substituted for G_o in Equation 10.85, we get

$$A_v(s) = \frac{G}{1 + G\gamma} = \frac{\left(\frac{G_o}{1 + s/\omega_o}\right)}{1 + \left(\frac{G_o}{1 + s/\omega_o}\right)\gamma} = \frac{G_o}{1 + s/\omega_o + G_o\gamma}. \quad (\text{Equation 10.87})$$

Note: I will try to be consistent, but in case I forget A_v means $A_v(s)$ (A_i means $A_i(s)$, etc.) if frequency dependence is involved.

If the dc gain, G_o , is very large we can assume that $G_o\gamma \gg 1$ for reasonable values of Z_A and Z_F . Equation 10.87 may then be reduced to

$$A_v(s) = \frac{G_o}{s/\omega_o + G_o\gamma} = \left(\frac{G_o}{\gamma G_o}\right) \left(\frac{1}{s/\gamma G_o\omega_o + 1}\right) = \frac{1}{\gamma(1 + s/\gamma G_o\omega_o)}. \quad (\text{Equation 10.88})$$

This gain equation has a scaling factor of $1/\gamma$ and contains a single pole at

$$\omega_c = \gamma G_o\omega_o = \frac{Z_A G_o\omega_o}{Z_A + Z_F} = \frac{G_o\omega_o}{1 + Z_A / Z_F}, \quad (\text{Equation 10.91})$$

where R_A and R_F would replace Z_A and Z_F if the circuit components were purely resistive instead of reactive. It can be shown that the bandwidth the inverting configuration using single-pole compensation is identical.

Mathematically we can show that, for large open-loop gains, the closed-loop gain bandwidth product is constant by

$$GBP = A_v\omega_c = \left(\frac{1}{\gamma}\right)(\gamma G_o\omega_o) = G_o\omega_o. \quad (\text{Equation 10.92, Modified})$$

So, for single-pole compensation, the gain bandwidth product is constant for the op-amp. The GBP tells us that as the closed-loop gain is increased, the bandwidth must decrease so that the product remains constant. This inverse relationship between gain and bandwidth may be an important tradeoff when designing op-amp circuits.

Phase Shift

For an ideal op-amp, there was no phase shift for the non-inverting configuration and the phase shift between input and output for the inverting configuration was determined to be 180° (since $\cos 180^\circ = -1$). For a practical op-amp, the phase shift is dependent on the frequency of the input signal. For example, an inverting configuration has a phase difference of 180° at dc. The angle will decrease as the frequency of the input signal increases due to the contribution of the pole(s) of the transfer function. At high frequencies, the phase difference approaches zero and a portion of the output signal is fed back to the input in phase with the input. This changes the feedback mechanism from negative to positive (which we will get into in the next section) and the amplifier may become unstable or marginally stable (in the marginally stable case, it exhibits behavioral characteristics of an oscillator).

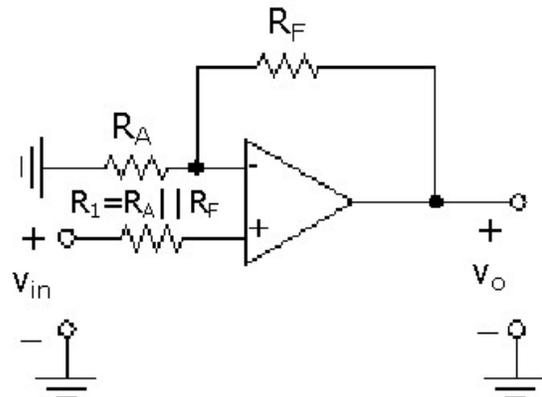
This potentially undesirable condition is avoided by internal frequency compensation or by including an external capacitor. In either case, since oscillation will not occur with positive feedback as long as the gain is less than unity, the strategy is to force the op-amp to have a gain of less than one at frequencies where the phase difference between input and output approaches zero.

Slew Rate

Ideally, the output of a non-inverting op-amp circuit would be a perfect step function for a step input. However, since the formation of a step input requires essentially infinite frequencies, and the practical op-amp has a response that is frequency dependent, we cannot realize an ideal output. This characteristic of op-amps, referred to as **slew rate limiting**, causes distortion of the output signal and is a figure of merit for the device.

Figure 10.30a (given to the right) illustrates the non-inverting configuration using purely resistive components and satisfying the bias balance condition. Equation 10.95 of your text is simply a repeat of Equation 10.88 and is given by

$$A_v(s) = \frac{v_o(s)}{v_{in}(s)} = \frac{1}{\gamma(1 + s/\gamma G_o \omega_o)}, \text{ where}$$



$$\gamma = \frac{R_A}{R_A + R_F} = \frac{1}{1 + R_F/R_A}, \quad (\text{Equation 10.95})$$

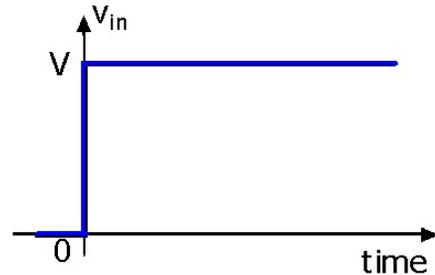
so that the low frequency gain, $1/\gamma = 1 + R_F/R_A$ and the gain bandwidth product is $G_o\omega_o$ as derived earlier.

If we apply the unit step function, as shown to the right and in Figure 10.30b, the input to the op-amp may be defined as

$$v_{in} = Vu(t),$$

where V is the magnitude of the input signal and the unit step, $u(t)$ is defined by

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$



Using Laplace transform methods, the input $Vu(t)$ becomes V/s , where s is the complex frequency. Solving Equation 10.95 for $v_o(s)$ and substituting in the expression for $v_{in}(s)$, we obtain

$$v_o(s) = \frac{v_{in}(s)}{\gamma(1 + s/\gamma G_o\omega_o)} = \left(\frac{V}{s}\right)\left(\frac{1}{\gamma}\right)\left(\frac{1}{1 + s/\gamma G_o\omega_o}\right). \quad (\text{Equation 10.96})$$

Using partial fraction expansion (if you have questions about this mathematical tool, let me know) and converting back to an expression in time, we get

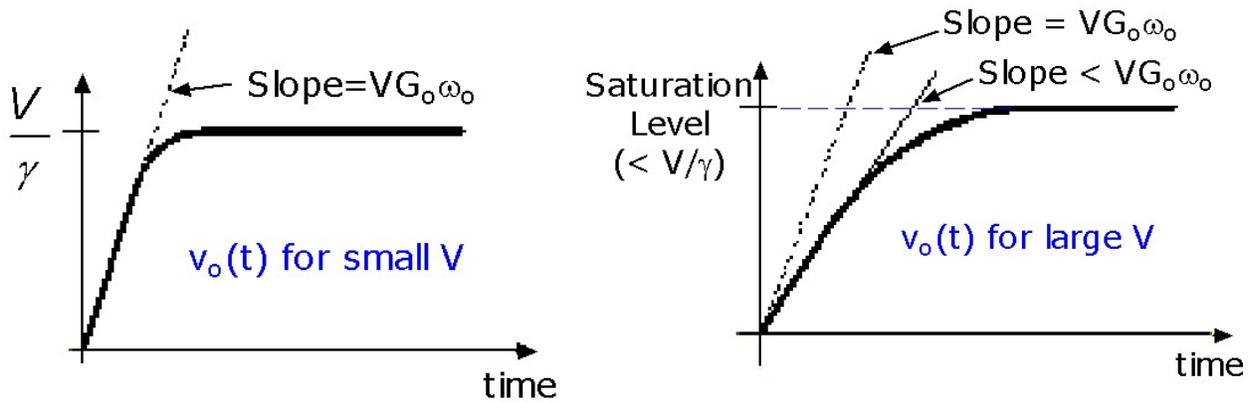
$$v_o(t) = \frac{V}{\gamma}\left(1 - e^{-\gamma G_o\omega_o t}\right). \quad (\text{Equation 10.97})$$

Recalling that the slope of a function is the derivative with respect to the independent variable (time in this case), we can define the initial slope of the output waveform (at $t=0$) to be

$$\frac{dv_o(t)}{dt} = \left(\frac{V}{\gamma}\right)\left(\gamma G_o\omega_o\right)e^{-\gamma G_o\omega_o t}\Big|_{t=0} = VG_o\omega_o. \quad (\text{Equation 10.98})$$

Note! This result, which is illustrated in Figure 10.30c (below and to the left), is valid for **linear operation only**. This means that the magnitude of the

input (V) must be sufficiently small so that the op-amp does not saturate. If the magnitude of the input is large enough to cause the op-amp to saturate, the output response will resemble Figure 10.30d (below and to the right). In Figure 10.30d, note that the initial slope of the output response curve is less than that predicted by the linear theory. The inability of the op-amp output to rise as fast as the linear theory predicts is defined as **slew rate limiting**.



In Figure 10.30d (above, right) the op-amp is said to be **slewing**, which occurs when the initial slope of the $v_o(t)$ response is less than $VG_o\omega_o$. When the op-amp is slewing, the initial slope of the output response is defined as the **slew rate**:

$$\text{Slew rate} = SR = \frac{dv_o(t)}{dt} (\text{maximum}). \quad (\text{Equation 10.99})$$

The slew rate for a particular op-amp is usually specified in the manufacturer's data sheets in volts per microsecond at unity gain. The slew rate is related to the **power bandwidth, f_p** , which is defined as the frequency at which a sinusoidal output, at rated output voltage, begins to exhibit distortion. Therefore, for a rated output voltage V_r and a slew rate of SR , the power bandwidth is found by

$$f_p = \frac{SR}{2\pi V_r}. \quad (\text{Equation 10.103})$$

If the output voltage is less than V_r , slew rate distortion begins at a frequency that is higher than f_p .

Note that all equations derived for Figure 10.30 may also be applied to a unity gain buffer by letting $R_A \rightarrow \infty$ since this will make $1/\gamma$ equal to one.

Designing Amplifiers Using Multiple Op-Amps

We now have all the tools to design amplifiers requiring more than one op-amp. In addition to the basic electrical characteristics of the op-amp, certain properties such as the input resistance specified for each terminal, bandwidth, output resistance, CMRR, PSRR, and the slew rate may be important in the selection of an appropriate device and the successful design of an amplifier system. Other considerations in the design of multiple op-amp circuits include:

- The gain per stage. This is determined by dividing the gain-bandwidth product (GBP) of the selected op-amp by the required bandwidth. This will also tell you the minimum number of gain stages required to meet any gain specification.
- Determine which inputs are negative and which are positive. This defines whether connections are made to the inverting or non-inverting terminal of the op-amp.
- If the required input resistance is larger than the value of the coupling resistor, the input voltage must be fed to the non-inverting terminal of the op-amp, since the input resistance to the inverting terminal is simply the value of the coupling resistance used (see Section G10 for specifics).
- At times, isolation of various sources in the circuit and phase relationships may become important.