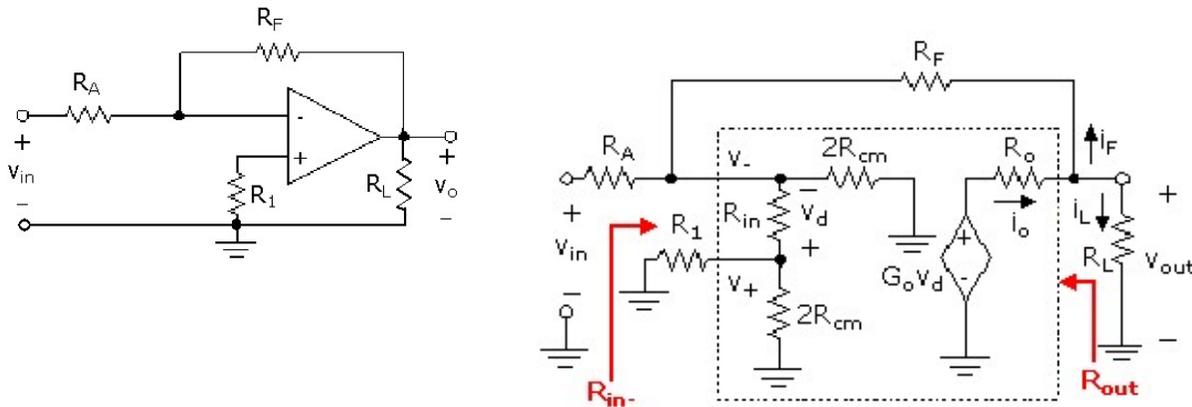


Section G9: Inverting Amplifier

We are now going to give the inverting op-amp configuration the same treatment as the non-inverting amplifier of the previous section. Specifically, more realistic input and output impedances, as well as voltage gain, will be developed for the modified op-amp model.

The schematic for a single input inverting amplifier is shown below and to the left, with the equivalent circuit given below and to the right (modified versions of Figures 9.36a and 9.36b of your text). Again, note that we are neglecting any input offset voltage and using the bias balance constraint ($R_1 \approx R_A || R_F$) discussed in G6 to negate any bias current effects in this equivalent circuit model.

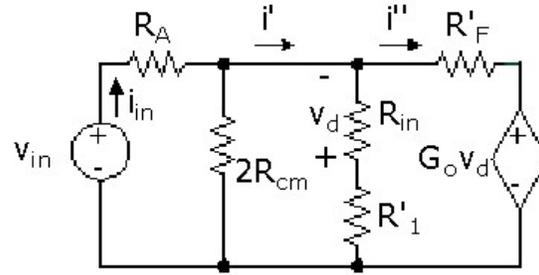


Input Resistance, R_{in-}

Just as we did for the non-inverting amplifier, to find the input resistance of the inverting configuration we need the equivalent resistance as seen by the input v_{in} as indicated in the figure above. We make the same observations as previously to simplify the above circuit. Specifically:

- $i_L \approx i_o$.
- $R_L \gg R_o$, so the output $v_{out} = G_o v_d$, and R_L is disregarded for analysis purposes.
- $R'_F = R_o + R_F$ when R_L is removed. *(Note: your text disregards R_o up front this time, but I'll keep it in for a while for consistency – we get the same place in the end!)*
- The resistances R_1 and $2R_{cm}$ are in parallel between the non-inverting terminal and ground, so we can define an equivalent resistance $R'_1 = R_1 || 2R_{cm}$.

These assumptions allow us to simplify the equivalent circuit to the representation to the right (Figure 9.37a). All the information we need is in this figure, so we don't need to repeat the multiple figures of the last section!



Realizing that $R_A \ll 2R_{cm}$, the voltage across $2R_{cm}$ is approximately equal to the input, v_{in} . Since the series combination of R_{in} and R'_1 is in parallel with $2R_{cm}$, this resistor combination also has a total voltage of v_{in} . Using a voltage divider,

$$v_d = \frac{-R_{in} v_{in}}{R_{in} + R'_1}, \quad (\text{Equation 9.71})$$

where the negative sign is introduced because of the defined polarities.

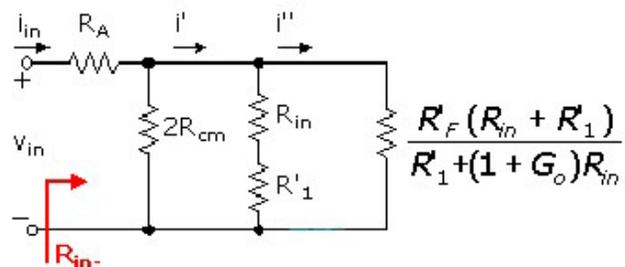
Writing a KVL around the rightmost loop (using the voltage across R_{in} and R'_1 equal to v_{in}),

$$v_{in} = i'' R'_F + G_o v_d = i'' R'_F - \frac{G_o R_{in} v_{in}}{R_{in} + R'_1}.$$

Rearranging, we can express v_{in} in terms of the current i'' and an equivalent resistance

$$v_{in} = \frac{i'' R'_F}{1 + \frac{G_o R_{in}}{R_{in} + R'_1}} = i'' \frac{R'_F (R_{in} + R'_1)}{R'_1 + R_{in} (1 + G_o)}.$$

The equivalent resistance of the rightmost loop is then included in a further simplification of the circuit as shown to the right (Figure 9.37b). If we analyze this equivalent resistance, the dominant term in the numerator is $R_{in} R'_F$ and the dominant term in the denominator is $G_o R_{in}$, so



$$\frac{R'_F (R_{in} + R'_1)}{R'_1 + R_{in}(1 + G_o)} \approx \frac{R'_F R_{in}}{R_{in}G_o} = \frac{R'_F}{G_o}.$$

If G_o is very large, the equivalent resistance becomes very small and the equivalent resistance of the three parallel legs becomes very small. For realistic values of G_o , the effective input resistance of an inverting amplifier is approximately equal to R_A , or

$$R_{in} = R_A + \frac{R'_F}{G_o} \cong R_A. \quad (\text{Equation 9.73})$$

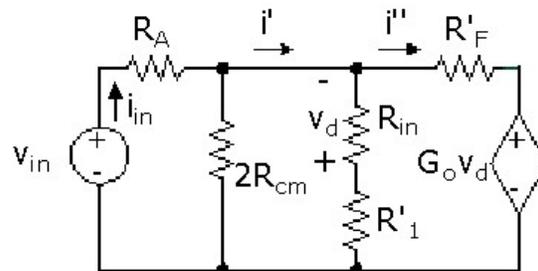
Output Resistance, R_{out}

As with the non-inverting configuration, the output resistance of the inverting configuration is found using the strategy of section G6. If all conditions and approximations are still valid (R_{cm} is so large that $R'_A = R_A || 2R_{cm} \approx R_A$, $R'_1 = R_1 || 2R_{cm} \approx R_1$, and $G_o R_A / (R_F + R_A) \gg 1$), the expression for the output resistance of the inverting configuration is also

$$R_{out} = \frac{R_o}{G_o} \left(1 + \frac{R_F}{R_A} \right) \quad (\text{Equation 9.52})$$

Voltage Gain, A_v

Figure 9.37a, repeated to the right, along with all the assumptions inherent in creating this figure from the original equivalent circuit, is used to define the voltage gain of the single input inverting configuration. Our task is to develop a relationship between the input, v_{in} , and the output $v_{out} = G_o v_d$. Of course, there is more than one way to do this, but we'll follow the strategy of your text and use a series of source transformations again.

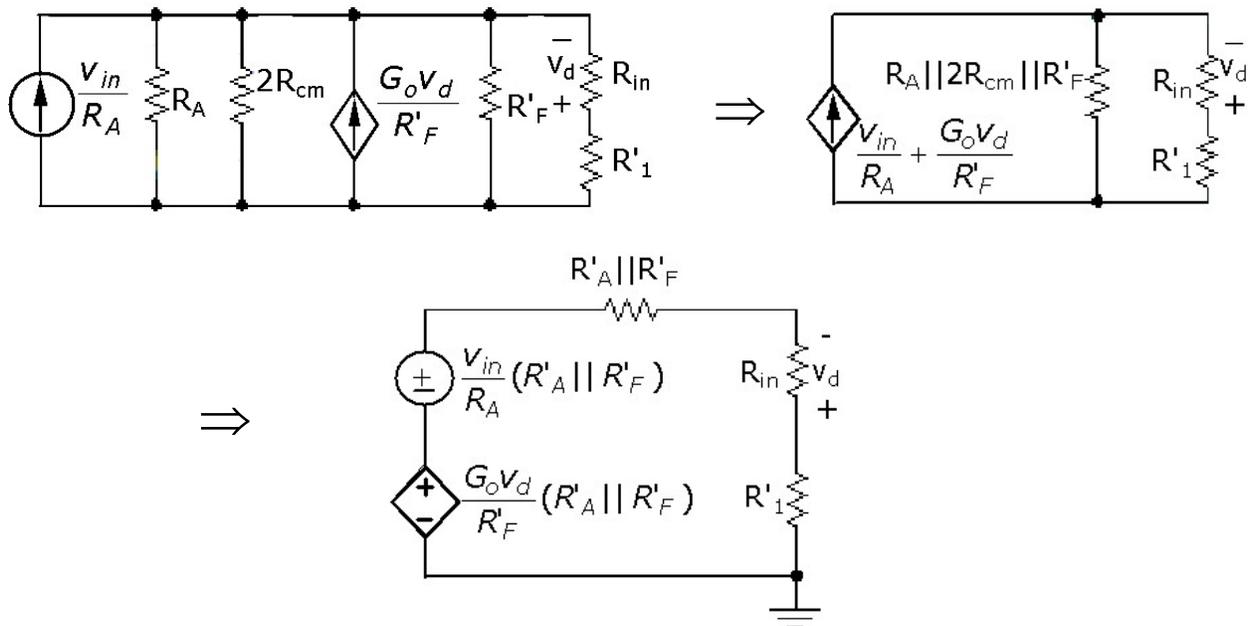


The sequence of steps illustrated in the figures below (Figure 9.38 of your text) is as follows:

1. Convert the independent voltage source/series resistor and dependent voltage source/series resistor into parallel current source/resistor combinations (independent and dependent, respectively).

- Combine the dependent and independent current sources into a single dependent source (note that Figure 9.38a in your text is incorrect). Combine parallel resistances into a single equivalent. Note that the R_{in} , R'_1 leg is left out of the parallel resistance combination since we want to keep v_d by itself.
- Use another source transformation to convert the parallel current source/resistor combination into series voltage source/resistor combinations. At this point, the independent and dependent sources are again separated – this is not necessary, but it keeps it clear what started independent and what started dependent. Also, in this last figure, the equivalent parallel resistance ($R_A || 2R_{cm} || R'_F$) has been relabeled $R'_A || R'_F$, where $R'_A = R_A || 2R_{cm}$.

Again, make sure you're comfortable with this and I'll meet you on the other side of the figures.



To calculate the voltage gain, we need to get a single equation in terms of v_{out} and v_{in} or, since $v_{out} = G_o v_d$, in terms of v_d and v_{in} . Using voltage division, we can find an expression for v_d :

$$v_d = \frac{-R_{in}}{(R'_A || R'_F) + R_{in} + R'_1} \left[(R'_A || R'_F) \left(\frac{v_{in}}{R_A} + \frac{G_o v_d}{R'_F} \right) \right]. \quad (\text{Equation 9.74})$$

Rearranging and collecting the terms containing v_d :

$$V_d \left[1 + \frac{R_{in} G_o (R'_A \parallel R'_F)}{R'_F [(R'_A \parallel R'_F) + R_{in} + R'_1]} \right] = \frac{-R_{in} (R'_A \parallel R'_F) V_{in}}{R'_A [(R'_A \parallel R'_F) + R_{in} + R'_1]}, \text{ or}$$

$$V_d = \frac{-R_{in} (R'_A \parallel R'_F) V_{in}}{R'_A [(R'_A \parallel R'_F) + R_{in} + R'_1]} \frac{R'_F [(R'_A \parallel R'_F) + R_{in} + R'_1]}{(R'_A \parallel R'_F) (R'_F + R_{in} G_o) + (R_{in} + R'_1) R'_F}$$

$$= \frac{-R_{in} R'_F (R'_A \parallel R'_F) V_{in}}{R'_A [(R'_A \parallel R'_F) (R'_F + R_{in} G_o) + (R_{in} + R'_1) R'_F]}$$

Using $v_{out} = G_o v_d$ and a bit of algebra, we can express the voltage gain for a single input inverting configuration as

$$A_{V-} = \frac{G_o V_d}{V_{in}} = \frac{-G_o R_{in} R'_F R'_A}{R'_A [R'_A R'_F + R'_A R_{in} G_o + (R'_A + R'_F) (R_{in} + R'_1)]}. \quad (\text{Equation 9.75})$$

If we now make the approximations that are allowed when R_{cm} is very large: $R'_A \approx R_A$ and $R'_1 \approx R_1$; as well as assuming $R_F \gg R_o$ so $R'_F \approx R_F$, and keep only the dominant terms, Equation 9.75 becomes

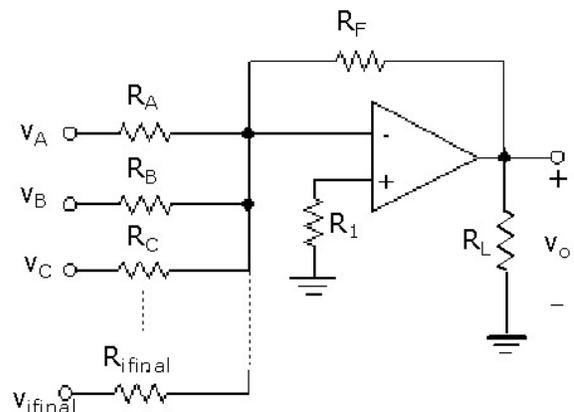
$$A_{V-} = \frac{-G_o R_{in} R_F}{G_o R_{in} R_A + R_{in} (R_A + R_F)}$$

If, in addition, G_o is very large, the first term in the denominator is dominant and the inverting voltage gain is

$$A_{V-} = \frac{-R_F}{R_A}, \quad (\text{Equation 9.76})$$

which is the voltage gain expression for the ideal op-amp in the single input inverting configuration! **Again, very cool!** This means that, just like for the non-inverting case, if our assumptions are valid (and again, if they're not, go check stuff), the gain of a practical device may be treated as ideal.

From section E3, we found that the output of a multiple input inverting amplifier as shown in the figure to the right may be expressed as a weighted sum of the inputs by



$$V_o = -\left(\frac{R_F}{R_A} V_A + \frac{R_F}{R_B} V_B + \frac{R_F}{R_C} V_C + \dots + \frac{R_F}{R_{ifinal}} V_{ifinal}\right) = -R_F \sum_{j=1}^{ifinal} \frac{V_j}{R_j} . \quad (\text{Equation 9.77})$$

To achieve bias balance, we choose

$$R_1 = R_F \parallel R_A \parallel R_B \parallel \dots \parallel R_{ifinal} . \quad (\text{Equation 9.78})$$

The output resistance for the multiple input inverting configuration is still given by Equation 9.52 where R_A is now replaced by $R_A \parallel R_B \parallel \dots \parallel R_{ifinal}$.