Section G8: Non-Inverting Amplifier

The schematic for a single input non-inverting amplifier is shown below and to the left, with the equivalent circuit given below and to the right. Again, note that we are neglecting any input offset voltage and using the bias balance constraint ($R_1 \approx R_A \parallel R_F$) discussed in G6 to negate any bias current effects in this equivalent circuit model. These illustrations are based on Figures 9.29a and 9.29b of your text.

**Input Resistance, $R_{in^+}$**

To find the input resistance of the non-inverting amplifier, we need the equivalent resistance as seen by the input $v_{in}$ as indicated in the figure above. To begin our simplifications of the equivalent circuit, we make the following observations:

- It is assumed (valid for normal operation) that $i_L \approx i_o$; that is, a much smaller fraction of the output current is fed back through $R_F$ than is supplied to the load.
- The output resistance ($R_o$) of the op-amp is usually much less than the load resistance ($R_L$), or $R_L >> R_o$. Note: If this inequality does not hold, the effective value of $R_o$ would be the parallel combination of $R_o$ and $R_L$, which, in turn, would reduce the effective gain of the amplifier. This observation allows us to neglect the voltage drop across $R_o$ and define the voltage across $R_L$ as $G_o v_d$. $R_L$ is no longer explicitly required and, for analysis purposes, may be removed from the simplified circuit.
- With $R_L$ removed from the equivalent circuit, $R_o$ and $R_F$ are in series. We may define an equivalent resistance $R'_F = R_o + R_F$.
- The resistances $R_A$ and $2R_{cm}$ are in parallel between the inverting terminal and ground. We may then define an equivalent resistance $R'_A = R_A \parallel 2R_{cm}$.
A KVL about the input loop indicates that $R_1$ is much less than either $2R_{cm}$ or $R_{in}$ (we will explicitly prove this shortly). This allows us to neglect $R_1$ (essentially replace with a short circuit) in the simplifications below.

With this information, the equivalent circuit (above, right) may be simplified to Figure 9.30a (below, left). I have added another step in the derivations to (hopefully) clarify what’s going on (I’ve also defined the nodes $v_+$ and $v_-$ in the first figure to show a “map”). Basically, in the sequence of figures below, we begin by incorporating our assumptions and observations; use a source transformation to turn a series voltage source/resistor combination into a parallel current source/resistor combination. We then combine parallel resistances and use another source transformation to turn the parallel current source/equivalent resistance into a series voltage source/equivalent resistance. See if it makes sense and I’ll catch you on the other side.

Ok, are you comfortable with all the graphical and mathematical gyrations?

The final step to this process is to determine the expression for $R_{in+}$. Looking at the last figure in the sequence above, note that the right hand loop is a series composed of $v_d=i'R_{in}$, $i'(R'_A||R'_F)$ and the dependent current source that is also a function of $v_d$, or (substituting $v_d=i'R_{in}$),

$$G_o v_d \left( \frac{R'_A}{R'_A+R'_F} \right) = i' G_o R_{in} \left( \frac{R'_A}{R'_A+R'_F} \right).$$

This substitution allows us to define a single expression to the right of $2R_{cm}$ as
Since the 2R\textsubscript{cm} resistor is in parallel with the input voltage \(v\textsubscript{in}\), we may define an equivalent resistance to the right of 2R\textsubscript{cm} to be

\[
\left[ (R\textsubscript{A}' \parallel R\textsubscript{F}') + R\textsubscript{in} \left( 1 + \frac{G\textsubscript{o} R\textsubscript{A}'}{R\textsubscript{A}' + R\textsubscript{F}'} \right) \right],
\]

as shown to the right and in Figure 9.30c in your text. The input resistance as seen by the source \(v\textsubscript{in}\) is

\[
R\textsubscript{in+} = 2R\textsubscript{cm} \left[ (R\textsubscript{A}' \parallel R\textsubscript{F}') + R\textsubscript{in} \left( 1 + \frac{G\textsubscript{o} R\textsubscript{A}'}{R\textsubscript{A}' + R\textsubscript{F}'} \right) \right], \quad \text{(Equation 9.55)}
\]

If we note that \(R\textsubscript{cm}, G\textsubscript{o}, \text{ and } R\textsubscript{in}\) are very much larger than \(R\textsubscript{A}\) and \(R\textsubscript{F}\), we may keep only the dominant terms and simplify Equation 9.55 to

\[
R\textsubscript{in+} \approx 2R\textsubscript{cm} \left[ \frac{G\textsubscript{o} R\textsubscript{A}' R\textsubscript{in}}{R\textsubscript{A} + R\textsubscript{F}} \right] = 2R\textsubscript{cm} \left[ \frac{G\textsubscript{o} R\textsubscript{in}}{1 + R\textsubscript{F}/R\textsubscript{A}} \right]. \quad \text{(Equation 9.56)}
\]

**Output Resistance, \(R\textsubscript{out}\)**

The output resistance of the non-inverting configuration is found using the strategy of section G6; i.e., applying a test voltage at the output, calculating the resulting current, and taking the ratio of voltage to current to obtain an equivalent resistance. Since all independent sources were set to zero in the derivation of section G6, we may use these results to define the output resistance of the non-inverting configuration as

\[
R\textsubscript{out} = \frac{R\textsubscript{o}}{G\textsubscript{o}} \left( 1 + \frac{R\textsubscript{F}}{R\textsubscript{A}} \right) \quad \text{(Equation 9.52)}
\]

**if all conditions and approximations are valid; i.e., if \(R\textsubscript{cm}\) is large enough that \(R\textsubscript{A}'=R\textsubscript{A}||2R\textsubscript{cm}\approx R\textsubscript{A}, R\textsubscript{1}'=R\textsubscript{1}||2R\textsubscript{cm}\approx R\textsubscript{1}, \text{ and } G\textsubscript{o} R\textsubscript{A}/(R\textsubscript{F}+R\textsubscript{A}) \gg 1.**
**Voltage Gain, \( A_{V+} \)**

The voltage gain for the single input non-inverting op-amp configuration is found by using a similar strategy to that for the input resistance. Your author performs a completely new derivation, but if we make the assumptions and observations as earlier, i.e.,

- \( i_L \approx i_o \)
- \( R_L >> R_o \), so \( v_{out} = G_o v_d \) and \( R_L \) does not need to be explicitly included,
- \( R'_F = R_F + R_o \) (if \( R_L \) is removed),
- \( R'_A = R_A \| 2R_{cm} \),
- \( R_1 \) is negligible. (Note: your author defines \( R'_1 = R_2 \| 2R_{cm} = (R_1 2R_{cm})/(R_1 + 2R_{cm}) \), and then uses the ratio \( R'_1 / R_1 \). Well, \( R'_1 / R_1 = 2R_{cm} / (R_1 + 2R_{cm}) \), which is very close to one if \( R_{cm} >> R_1 \) – and it will be, so let’s just skip that part!),

Now, we can simplify the original equivalent circuit to the figure at the right (Note that this is the same as Figure 9.30c, except now the voltage across the \( 2R_{cm} \) resistor is explicitly defined). Using our previous results, we may now write a KVL and solve for the current \( i' \) as

\[
v_{in} = i' \left[ (R'_A \| R'_F) + R_{in} \left( 1 + \frac{G_o R'_A}{R'_A + R'_F} \right) \right]
\]

\[
i' = \frac{v_{in}}{(R'_A \| R'_F) + R_{in} \left( 1 + \frac{G_o R'_A}{R'_A + R'_F} \right)}
\]

Recall that \( v_d = i'R_{in} \) and we have

\[
v_{out} = G_o v_d = G_o i'' R_{in} = \frac{v_{in} G_o R_{in}}{(R'_A \| R'_F) + R_{in} \left( 1 + \frac{G_o R'_A}{R'_A + R'_F} \right)}
\]

The voltage gain, \( A_{V+} \), of the single input non-inverting configuration is equal to
\[ A_{v+} = \frac{V_{out}}{V_{in}} = \frac{G_o R_{in}}{(R'_A \parallel R'_F) + R_{in} \left( 1 + \frac{G_o R'_A}{R'_A + R'_F} \right)} , \]

which is the same as Equation 9.64 of your text except for the \( R_1 \) and \( R'_1 \) terms. The \( R'_1/R_1 \) ratio we took care of earlier, and it turns out that the \( R'_1 \) term in the denominator of Equation 9.64 is also negligible. If we retain only the dominant terms of the above expression for \( A_{v+} \) (or Equation 9.64) and let the gain term \( G_o \) become very large, we have

\[ A_{v+} = \frac{G_o R_{in}}{R_{in} \left( 1 + \frac{G_o R'_A}{R'_A + R'_F} \right)} \approx \frac{G_o R_{in}}{R_{in} G_o R'_A} \approx \frac{R'_A + R'_F}{R'_A} . \]

Further if we note that \( R'_A \approx R_A \) if \( R_{cm} \gg R_A \) and \( R'_F \approx R_F \) if \( R_F \gg R_o \), we get

\[ A_{v+} = \frac{R_A + R_F}{R_A} , \quad \text{(Equation 9.66)} \]

which is just the gain expression for a single input ideal op-amp in the non-inverting configuration – cool! This means that if our assumptions are valid (and if they’re not, we’ve got bigger problems), the gain of a practical device may be treated as ideal.

From section E3, we found that the output of a multiple input non-inverting amplifier as shown in the figure to the right may be expressed as a weighted sum of the inputs by

\[ v_o = \left( 1 + \frac{R_F}{R_A} \right) (R_1 \parallel R_2 \parallel R_3 \parallel \cdots \parallel R_{n_{final}}) \left( \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \cdots + \frac{v_{n_{final}}}{R_{n_{final}}} \right) \]

\[ = \left( 1 + \frac{R_F}{R_A} \right) (R_1 \parallel R_2 \parallel R_3 \parallel \cdots \parallel R_{n_{final}}) \sum_{k=1}^{n_{final}} \frac{v_k}{R_k} \]  

(Eqn 9.68)

The output resistance for a multiple input amplifier remains the same as is given by Equation 9.52 since it does not specifically depend on any input.
Finally, we may satisfy the bias balance constraint by choosing resistors such that the following equality holds

\[ R_A \parallel R_F = R_1 \parallel R_2 \parallel R_3 \parallel \ldots \parallel R_{\text{final}}. \]  

(Equation 9.69)