Section G10: Differential Summing

In Section E3, we introduced the ideal op-amp with combined inverting and non-inverting inputs and got an expression for the output to be

\[
v_o = \left(1 + \frac{R_F}{R_A || R_B || R_C || \ldots || R_{ifinal}}\right) \left[\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \ldots + \frac{v_{ifinal}}{R_{ifinal}}\right] - \left(\frac{R_F}{R_A} v_A + \frac{R_F}{R_B} v_B + \frac{R_F}{R_C} v_C + \ldots + \frac{R_F}{R_{ifinal}} v_{ifinal}\right).
\]

We’re going to do the same thing here, with the same figure (to the right, a modified version of Figure 9.40 in your text), but now we’re going to make it more “technical.” This is officially a weighted sum of multiple inputs where the sum includes both positive and negative signs, or **differential summing**. Note also that we have now included \(R_X\) and \(R_Y\), resistors that provide a path to ground from the non-inverting and inverting terminals respectively. By defining an equivalent parallel resistance \(R_{eq}\) to be

\[
R_{eq} = R_A || R_B || R_C || \ldots || R_{ifinal} || R_Y,
\]

we may rewrite the expression above for the output voltage as

\[
v_o = \left(1 + \frac{R_F}{R_{eq}}\right) \left[\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \ldots + \frac{v_{ifinal}}{R_{ifinal}}\right] - \frac{R_F}{R_{ifinal}} \sum_{j=A}^{ifinal} \frac{v_j}{R_j}.
\]

(Equation 9.90, Mod.)

To achieve **bias balance**, resistors are chosen such that

\[
R_1 || R_2 || R_3 || \ldots || R_{ifinal} || R_X = R_F || R_A || R_B || \ldots || R_{ifinal} || R_Y.
\]

(Equation 9.92)

If there is a single inverting input and a single non-inverting input, as illustrated to the right (a modified version of Figure 9.41 in your text), the configuration is known as a **differencing**.
amplifier. Using Equation 9.90, and determining the input at the non-inverting terminal by a voltage divider, we obtain

\[ v_o = \left( 1 + \frac{R_F}{R_A} \right) \frac{R_X v_1}{R_1 + R_X} - \frac{R_F}{R_A} v_A. \]  

(Equation 9.93)

To satisfy the bias balance constraint, resistors are chosen such that

\[ R_1 || R_X = R_A || R_F. \]  

(Equation 9.94)

The input resistance seen by \( v_A \) is \( R_A \), while the input resistance seen by \( v_1 \) is calculated by \( R_{in1} = R_1 + (R_X || R_{in+}) \), where \( R_{in+} \) is the input resistance of the non-inverting configuration discussed in section G8, or

\[ R_{in+} \approx 2R_{cm} \left( \frac{G_o R_{in}}{1 + R_F/R_A} \right). \]  

(Equation 9.56)

Since, for the practical case, \( R_{in+} >> R_X \), the input resistance seen by \( v_1 \) is approximately equal to \( R_1 + R_X \). The output resistance is still given by Equation 9.52,

\[ R_{out} = \frac{R_o}{G_o} \left( 1 + \frac{R_F}{R_A} \right). \]  

(Equation 9.52)

To achieve unity gain differencing, where the output is given by \( v_o = v_1 - v_A \), set \( R_A = R_F = R_1 = R_X = R \). Using the expressions above, this results in an input resistance seen by \( v_A \) of \( R \) and an input resistance seen by \( v_1 \) of \( 2R \), and the output resistance is still given by Equation 9.52.

If equal gain differencing is desired for a gain other than one (unity), the circuit resistors are defined by \( R_1 = R_A \) and \( R_X = R_F \). The output voltage (using Equation 9.93) becomes

\[ v_o = \frac{R_F}{R_A} (v_1 - v_A). \]  

(Equation 9.97)

Using the same strategy as earlier, the input resistance seen by \( v_1 \) is approximately \( R_A + R_F \) since \( R_{in+} >> R_F \). The input resistance seen by \( v_A \) is \( R_A \) and the output resistance is given by Equation 9.52... yet one more time!
A useful modification of the differencing amplifier configuration involves the addition of a single pole double throw (SPDT) switch and results in the sign switcher shown to the right (based on Figure 9.42a in your text). Depending upon the position of the switch, two possible outcomes may be realized:

- If the switch is tied to ground, we have a simple single input inverting amplifier configuration where $v_A = v_{in}$. If $R_F = R_A = R$, $v_o = -v_{in}$.
- If the switch is tied to $v_{in}$, we have a differencing amplifier configuration where $v_A = v_1 = v_{in}$. If $R_F = R_A = R$, $v_o = 2v_{in} - v_{in} = v_{in}$ (using Equation 9.93 where $R_X \to \infty$).

The input resistance is $R$, and the bias is balanced, for each position of the switch.

A modified version of Figure 9.42b is given to the right and illustrates an alternative configuration for the sign switcher. In this case, a single pole single-throw (SPST) switch is utilized. As for the previous circuit, depending on the switch position, we may get one of two possible outputs:

- If the switch is open, we have a differencing amplifier configuration where $v_A = v_1 = v_{in}$. If $R_F = R_A = R$, $v_o = v_{in}$ ($= 2v_{in} - v_{in}$). Note that having the series connection of $R_1$ and $R_2$ in the non-inverting leg does not matter when calculating gain.
- If the switch is closed, the non-inverting terminal is tied to ground through $R_2$. What remains is a single input inverting amplifier and, if $R_F = R_A$, $v_o = -v_{in}$.

Although we get the same results at the output for both configurations, this implementation results in unequal bias and input resistances for the two possible SPST switch positions.