Section E4: (Some) Other Op-Amp Applications

In the previous section, we concentrated on using the op-amp to create an output that was a linear combination of single or multiple inputs applied to the inverting and/or non-inverting terminals. In this last section of our introduction to operational amplifiers, we’re going to briefly discuss some other cool things this IC can do.

**Negative Impedance Circuit**

Up to now, you’ve been told that there is no such thing as a negative resistance, right? Well, it turns out that the op-amp can be configured in such a way that it essentially cancels out unwanted positive resistances (or impedances) by producing what looks like a negative input resistance (or impedance).

The negative impedance circuit of Figure 2.17 is reproduced to the right. In this figure, $R_A$ is the equivalent resistance at the inverting terminal; i.e., $R_A = R_a || R_b || ... || R_{\text{final}}$. Following our analysis procedure for ideal op-amps:

*Write the KCL equation at the inverting terminal, $v_-$, using $i_-=0$.*

$$\frac{0-v_-}{R_A} + \frac{v_a - v_-}{R_F} = i_- = 0$$

*Write the KCL equation at the non-inverting terminal, $v_+$, using $i_+=0$.*

$$i_in + \frac{v_a - v_+}{R} = i_+ = 0, \text{ where } v_+ = v_{in}$$

*Set $v_- = v_+$ and solve for the desired closed-loop gains (or resistor values or whatever the problem is asking for).*

In this case, we’re interested in $R_{in}$, where $R_{in} = v_{in}/i_{in}$. Substituting $v_+$ for $v$ in the first equation above and solving both equations for $v_o$,

$$v_o = \left(1 + \frac{R_F}{R_A}\right)v_{in}, \quad v_o = v_{in} - R_i n$$
Setting these two expressions for $v_o$ equal to each other and collecting terms:

$$v_o = \left(1 + \frac{R_F}{R_A}\right)v_{in} = v_{in} - Ri_{in} \quad ; \quad \frac{R_F}{R_A}v_{in} = -Ri_{in}$$

we are ready to define the input resistance, $R_{in}$:

$$(\text{Equation 2.47})$$

$$R_{in} = \frac{v_{in}}{i_{in}} = -\frac{RR_A}{R_F}.$$ 

So, by providing feedback from the output to both the inverting and non-inverting terminals, a negative resistance may be developed. Note that if any or all of the external components are impedances instead of simple resistances, a negative impedance may be developed.

**Dependent Current Generator**

A slight modification of the negative impedance circuit above, in which we set the feedback resistor equal to the equivalent resistance applied to the inverting terminal (i.e., $R_F=R_A$), allows us to create a circuit that produces a load current that is directly proportional to the applied input, but independent of the load resistance.

Using the results of Equation 2.47 and substituting $R_F=R_A$, the input resistance of the portion in the dashed box of the figure above (Figure 2.18a in your text) becomes simply $-R$. Redrawing this figure with the dashed box replaced by $-R$, we get the equivalent circuit shown to the right (Figure 2.18b in your text). Defining an equivalent resistance and solving for $i_{in}$, we get
\[ R_{eq} = R + (-R \parallel R_L) = R + \frac{-RR_L}{R_L - R} = \frac{-R^2}{R_L - R}; \quad i_n = \frac{v_{in}}{R_{eq}} = \frac{v_{in}(R_L - R)}{-R^2}. \]

Expanding the circuit back out and using the current divider rule to solve for the load current \( i_L \):

\[ i_L = \frac{(-R)i_n}{R_L + (-R)} = \left( \frac{-R}{R_L - R} \right) \left( \frac{R_L - R}{-R^2} \right) v_n = \frac{v_{in}}{R}. \]

(Equation 2.49)

So, by letting \( R_F = R_A \) in the negative impedance circuit, the load current is independent of the actual load and is directly proportional to the input voltage through the resistance \( R \). This behavior is known as a **voltage-to-current converter**, or a **current generator**.

A simpler version of the voltage-to-current converter is shown to the right (a modified version of Figure 2.20 in your text). The output voltage, \( v_o \), is \( i_L R_L \). Using our ideal op-amp analysis techniques (that should be **very** familiar by now!), we get

\[ v_+ = v_{in} = v_- \]
\[ v_o = i_L R_1 = v_- = v_{in}. \]

and, solving for the load current, \( i_L = \frac{v_{in}}{R_1} \).

Although this circuit once again produces a current that is directly proportional to the input and independent of the load resistance, there is a practical concern with this configuration. Note that, unlike the first current generator circuit presented, neither end of the load resistor is grounded. This is known as a floating ground and may result have undesirable effects on the output signal.

A third incarnation of the voltage-to-current converter that allows grounding of the load resistor is shown to the right but requires two inputs, \( v \) and \( v_{in} \) (Figure 2.21 in your text).
Analyzing the circuit by writing KCL equations at \( v_1 \) and \( v_2 \), with currents going into the node defined as positive:

\[
\frac{v - v_+}{R_1} + \frac{v_+ - v_-}{R_2} = 0; \quad v_- = \frac{R_2 v + R_1 v_0}{R_1 + R_2}
\]

\[
\frac{v_{in} - v_+}{R_1} + \frac{v_+ - v_+}{R_2} - i_L = 0; \quad v_+ = \frac{R_2 v_{in} + R_1 v_0 - R_1 R_2 i_{in}}{R_1 + R_2}
\]

Setting \( v_+ = v_- \), canceling the common denominator and the \( R_1 v_0 \) term, we get \( R_2 v = R_2 v_{in} - R_1 R_2 i_{in} \). From this simplified expression, we can derive the expression for the load current:

\[
i_L = \frac{v_{in} - v}{R_1}. \quad \text{(Equation 2.54)}
\]

So... we have again removed any dependence of the load current on the load resistance. The distinction of this circuit configuration is that we now have a load current that is proportional to the difference between two inputs. Note that if \( v = 0 \), we’re back to the first circuit we looked at.

**OK, that’s enough of this function. Let’s move on...**

**Current-to-Voltage Converter**

We’ve done the voltage-to-current converter way past enough, so let’s go the other way for a while. The circuit of Figure 2.19 in your text, and reproduced to the right, is defined as a **current-to-voltage converter** by your author. Note that this is a simple **inverting amplifier with unity gain** from the previous section of study.

Analyzing this circuit by using \( i_{in} \) instead of \((v_{in} - v_+)/R \) at the inverting terminal, we can see that the output voltage is directly proportional to the input current through the resistance \( R \), or

\[
v_o = -i_{in} R.
\]