

Section C3: BJT Equivalent Circuit Models

OK, we've got the terminal currents defined in terms of our gain constants and each other. Now... we've got to come up with a model for the entire device that we can put in an electrical circuit for design and analysis!

The large signal model introduced in section 4.2 of your text (and illustrated in Figure 4.3) is known as the Eber-Molls Model. This model is the basis for PSpice simulations and is used for large input swings. However, for now, we're going to be restricting our work to the **small signal region**.

Small signal operation is pretty much what it sounds like – we are going to restrict the operation of the transistor to a small variation about a defined operating point – a range over which the exponential curves we've been talking about are **almost linear**. This allows us to develop workable models using the strategy of "linearizing" nonlinear relationships – a powerful tool and serious sanity-saver if done correctly!

Remember that for a forward-biased pn junction (a diode), the current-voltage relationship is exponential and is repeated below:

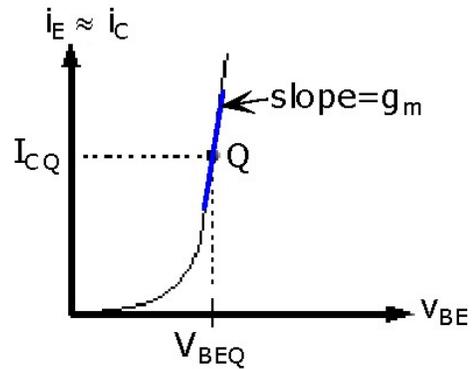
$$i_D = I_0 e^{\frac{v_D}{nV_T}} \quad \text{(Equation 3.27)}$$

We can use the above equation, along with what we've learned about the BJT and notation conventions to get where we want to be as follows:

- For the operational mode we're going to be concentrating on (normal active), the emitter-base junction (EBJ) is forward biased, so v_{BE} will replace v_D in Equation 3.27.
- v_{BE} is the total voltage across the junction and may consist of an ac and a dc portion, so $v_{BE} = V_{BE} + v_{be}$.
- The current associated with the EBJ would be i_E (carrier flow from emitter into the base), but using the approximations of Equation 4.9, we can substitute i_C (that still has an ac part and a dc part).
- Using mathematical tricks (aka "techniques") used in your text, the resulting mess can be simplified... and
- By defining a parameter known as **transconductance** (g_m , in units of S or Ω^{-1}), and keeping only the ac part, Equation 4.12 is derived

$$i_c = g_m v_{be} \quad \text{(Equation 4.12)}$$

If anyone has questions about the mathematical derivation, or the meaning of the transconductance in terms of a partial derivative, let me know and I'll be happy to go over it. Otherwise, we're just going to take it on faith and recognize the transconductance parameter as the slope of the i_C - V_{BE} curve evaluated at a defined dc operating point (the Q-point from our diode days) as shown in Figure 4.4(a) to the right.



Defining a resistance as the inverse of the transconductance parameter, we can introduce the **emitter resistance parameter** r_e . r_e is the resistance to the ac part of the emitter current as it moves through the EBJ, and is actually responsible for creating the ac voltage component v_{be} . Note: This is also referred to as the dynamic resistance (r_d) in Equation 4.19 of your text – once again harking back to our diode days.

So, to recap:

1. The emitter (and therefore collector) current is a function of the total base-emitter voltage (ac+dc);
2. the transconductance (g_m) is a measure of this relationship (Equation 4.12);
3. the emitter resistance (r_e) is $1/g_m$; and
4. r_e and the ac part of emitter current get together to create v_{be} ...which is part of step one; and
5. It all depends on the dc part (the Q-point)!

Whew! Wanna keep going? (Like you have a choice, right?)

The definition of the emitter resistance in terms of the transconductance, and the relationship between emitter and base currents (Equation 4.8, $i_E = \beta i_B$) allows us to define an important parameter, r_π (you'll see where the π comes from in a minute). Since the ac part of the EBJ voltage must be the same whether you're "looking" into the emitter or "looking" into the base,

$$v_{be} = i_e r_e = \beta i_b r_e = r_\pi i_b.$$

r_π is the ac resistance associated with the base current, and is approximately β times larger than the emitter resistance r_e , or (recalling that $r_e = 1/g_m$)

$$r_{\pi} = \beta r_e = \frac{\beta}{g_m} \quad \text{(Equation 4.14)}$$

The above derivation illustrates a practical tool known as **impedance reflection** – that is, if a resistance is in the emitter, it “looks” β times larger to the base. Equivalently, a resistance in the base, “looks” β times smaller to the emitter ($r_e = r_{\pi}/\beta = g_m r_{\pi}$)... trust me, this will be used!

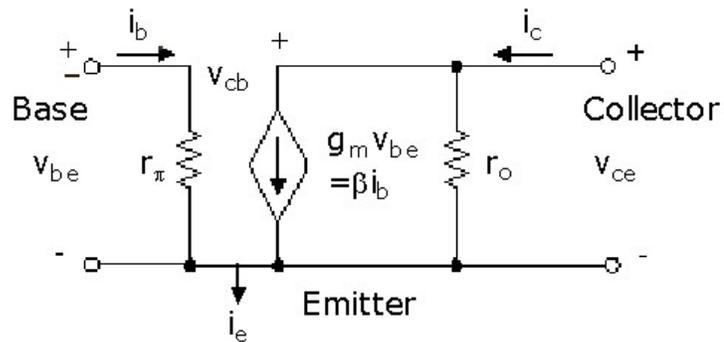
This all has to do with the current relationships of the device and the fact that voltage must be constant. For the voltage across the emitter resistance r_e in the emitter circuit, $v = r_e i_E = r_e \beta i_B$ (using the approximation $\beta \gg 1$). However, if we are looking at the equivalent resistance in the base circuit, we use the value of r_{π} , where $r_{\pi} = \beta r_e$. The effective voltage will remain the same since $i_B = i_E/\beta$. Again, many words that may be summarized as

$$V_{\pi} = V_{be} = i_B r_{\pi} = i_E r_e.$$

Impedance reflection is an extremely important concept that we will be using a whole lot! If you are not comfortable with this concept after chewing on it for a while, let me know. I guarantee that you will not be the only one!

The Hybrid- π Model

Keeping in mind the dependencies we have defined, and realizing that the base-emitter and collector-emitter curves of Figure 4.4 are nonideal (this will be discussed in detail in Section C4, but for right now - they possess some finite, non-zero slope



and therefore there is some resistance associated with the junctions), the **hybrid- π model of a BJT in the common-emitter configuration** is presented in Figure 4.5 (above). The common-emitter designation is used since the emitter terminal is “common” to both the input and output port. We will be discussing this configuration, as well as the common-base and common-collector, in a little while.

The hybrid- π model is extremely useful for BJT amplifier design and analysis, but it should be noted that it is restricted to **ac small-signal, low- to mid-**

band frequency use. The frequency limitation will become clearer in the next section, when we talk about the design and analysis of BJT amplifiers.

Don't worry; it's actually not that bad. With practice, you will see that relevant parameters are defined or easily derived and we will be able to use them effectively.

Two-Port Small Signal ac Models

Another valuable model of the bipolar junction transistor is derived using two-port network theory. Two-port models are based on the concept of an input port and an output port, each of which contains two terminals. In this fashion, the device may be modeled as a "black box" with internal characteristics defined by the voltage and current characteristics of the input and output terminals. This concept is illustrated in Figure 4.6(a) of your text for a common-emitter configuration of the BJT.

The most common two-port network model for electronic applications is the **h-parameter model** illustrated in Figure 4.6(b) of your text. Comparing the h-parameter and hybrid- π models, the relationships of Equations 4.15 through 4.17 are developed. Although we will very rarely be working exclusively with h-parameters in this course, many times characteristics of interest are presented in this fashion on spec sheets and knowing where to find the relationships may be a stress-saver! Of particular benefit is Equation 4.18, which approximates the relationship between the hybrid- π and h-parameter model characteristics:

$$r_{\pi} \approx h_{ie} = (\beta + 1)r_e \approx \beta r_e$$

$$g_m = r_e^{-1} = \frac{\beta + 1}{h_{ie}} \approx \frac{\beta}{h_{ie}}$$

$$\beta i_b \approx g_m v_{be}$$

$$r_o \approx \frac{1}{h_{oe}}$$

(Equation 4.18)

BJT Models for Computer Simulations

Electronic simulation and circuit analysis programs employ sophisticated models to describe and predict the behaviors of active devices. Since computers are ideally suited to crunching massive quantities of numbers, models for some active devices include *WAY* more parameters than we would ever attempt to keep track of by hand! The complexity of computer

modeling comes from the attempt to accurately portray the nonlinear characteristics of the device.

An example of the 40 parameters used in PSPICE for a generic BJT is illustrated in Table 4.1 of your text, where any or all of these parameters may be controlled on the .MODEL statement of the command file. However, most of the time in our studies, the majority of these parameters will be set to their default values – a very good thing!

We're going to be using simulation throughout our studies, both to verify hand analysis and design, and to generate plots of transient, ac, dc, and frequency responses. Examples will contain copious comments to (hopefully) clarify the command file structure. There are several links in the PSpice Resources link in WebCT that provide more information and resources on SPICE and/or MICROCAP. This list is by no means complete, and if you find anything else that looks good – please let me know so I can add it.