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Experimental Results in Bearings-only Tracking using the Sequential Monte-Carlo Probability Hypothesis Density Filter

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ABSTRACT

We evaluate the use of a probability hypothesis density (PHD) filter in a bearings-only tracking application. The main feature of a PHD filter is that it propagates the first-order statistical moment of a multisource posterior distribution. Multisource estimation using a PHD filter has been shown to reliably track multiple simulated targets in the bearings-only case. In this paper we evaluate the utility of the sequential Monte-Carlo PHD filter for tracking surface ships using bearings-only data acquired from a Bluefin-21 unmanned underwater vehicle in Boston Harbor. The unmanned underwater vehicle was equipped with a rigidly mounted planar hydrophone array that measures the bearing angle to sources of acoustic noise, of which shipping traffic is the dominant source. We further evaluate several target maneuvering models, including clockwise and counter-clockwise coordinated turns. The combination of the coordinated turn models with a constant velocity model is used in a multiple model PHD filter. The results of the multiple model PHD filter are compared to the results of a PHD filter using only a constant velocity model.

Keywords: Multiple Model, Probability Hypothesis Density, Bearings-only tracking

1. INTRODUCTION

Our aim is to evaluate the probability hypothesis density (PHD) filter in tracking applications using data from field trials in which an unmanned underwater vehicle (UUV) measures relative bearing to a surface support ship.\textsuperscript{1} Our objective is for the UUV to estimate the trajectory of the surface support ship using a sequence of relative bearing measurements. In this application, one of the main challenges is port/starboard ambiguity of the bearing sensor. That is, unique noise sources on the port or starboard sides of the UUV may appear identical to the bearing sensor. A PHD filter is used to estimate both port and starboard potential tracks corresponding to each ship in the area of operation. For certain classes of UUV trajectories, a sequence of bearing measurements correspond to a unique target track.\textsuperscript{2} UUV maneuvers such as occasionally turning to port or starboard can be sufficient to eliminate port/starboard ambiguity. Measurements obtained from other ships in the area are treated as noise. Three tracking scenarios are examined in which the support ship: remains stationary, travels on a north/south trajectory with 180 degree turns, or follows a box pattern trajectory centered around the UUV. The latter two scenarios require that the PHD filter estimate the position of a maneuvering target. An extension to the PHD filter that embeds multiple models of target motion to address the case of a moving target is used. Much of the work found in literature deals with simulated non-maneuvering targets.\textsuperscript{3–5} However, our work aims to evaluate the performance of the PHD filter with realistic target behavior.

Multiple model (MM) filters are often used to estimate the trajectory of a single maneuvering target. Multiple model approaches typically use a family of candidate motion models and apply some procedure to select the model that reflects the target’s motion, typically an a priori Markov transition matrix that assigns a fixed probability for transitioning between motion models at each time-step. The interacting multiple model extended Kalman filter (IMM EKF) uses a separate EKF for each motion model, and propagates the target state as a Gaussian mixture.\textsuperscript{6} The IMM EKF typically uses three motion models (straight line motion and clockwise/clockwise-turn motion models). The goal of the IMM EKF is to merge the three motion components such that their first and second-order statistical moments match. However, the IMM EKF is susceptible to track divergence as it relies on two sources of approximation: approximating nonlinear transformations by linearizing about some point.
of operation, and approximating an exponentially growing number of Gaussian mixtures as a finite Gaussian mixture.

Several particle-based methods, such as the jump-Markov system particle filter\(^7\) (JMSPF), auxiliary multiple model particle filter\(^8\) (AMMPF) and multiple model particle filter\(^9\) (MMPF), have shown to suppress track divergence caused by a maneuvering target. In cases where a target’s turn-rate is unknown and not part of the augmented target state, particle methods typically require a priori knowledge of target behavior, that is, typical turn maneuver accelerations are needed to estimate the targets turn-rate given estimates of the target’s velocity.\(^10\)–\(^12\) The MMPF augments the target’s state with a discrete random variable that denotes which motion model best explains the measurements (relative bearing angle) of the target’s motion. Particles that survive resampling estimate the target state and motion model that best reflects the target’s motion. This is because the probability of a target following a particular motion model becomes more prevalent than others. The AMMPF is a hybrid of the auxiliary particle filter and the multiple model particle filter. The AMMPF attempts to characterize the joint probability density function (pdf) of the target, each particle, and motion model given a sequence of measurements. The pdf is marginalized to obtain the target density given the sequence of measurements. The MMPF and the AMMPF are shown to have similar performance.\(^9\),\(^13\) The JMSPF differs from the other particle methods in that rather than using a particle filter to estimate target state estimates, the particle filter is used to estimate the sequence of motion models that reflect target behavior given a sequence of measurements. The target state estimates are computed using a nonlinear filter—typically an EKF. Thus, the measurement conditioned joint target/model density is factorized as the product of the model sequence and state estimate densities. The JMSPF is shown to be less effective than the MMPF and AMMPF because the EKF approximation of the state hinders its performance.\(^9\),\(^13\)

An extension of the multiple model approach for tracking multiple maneuvering targets is presented in Ref. 14. The jump-Markov PHD filter is derived and implemented through Gaussian mixtures. The authors address two cases: linear Gaussian jump-Markov multitarget models for when a target’s turn-rate is known a priori; and nonlinear Gaussian jump-Markov multitarget models for when a target’s turn-rate is not known a priori. The former implementation augments the target state vector with an unknown turn-rate that is estimated through the PHD filter recursion. We present an extension of the sequential Monte-Carlo (SMC) PHD filter to arrive at an alternative multiple model probability hypothesis density (MMPHD) that is a multitarget extension of the filter proposed in Refs. 9, 13 and evaluate its effectiveness using real-world data.

This paper is organized as follows. Section 2 introduces the problem, Section 3 reviews background material on the probability hypothesis density. Multitarget tracking, the probability hypothesis density filter, and the multiple model extension are presented in Section 4. Finally, Section 5 and 6 presents implementation results and concluding remarks, respectively.

### 2. PROBLEM FORMULATION

The aim of this work is to estimate a target’s state, \(x = [\xi \eta \dot{\xi} \dot{\eta}]^T\), consisting of position, \((\xi, \eta)\), and respective velocities, \((\dot{\xi}, \dot{\eta})\), given a sequence of relative bearing measurement, \(\{z_1, z_2, ..., z_k\}\). A user-defined motion model is used to model the evolution of the target’s state. In filtering techniques, the motion model is typically used as the transitional prior and thus, affects the performance of the estimator.\(^15\) Most motion models that describe target behavior on a two-dimensional horizontal plane can be written in the form of the standard curvilinear-motion model\(^16\)

\[
\begin{align*}
\dot{\xi}(t) &= v(t) \cos \phi(t) \\
\dot{\eta}(t) &= v(t) \sin \phi(t) \\
v(t) &= a(t) \\
\dot{\phi}(t) &= a_n(t)/v(t)
\end{align*}
\]

The operator, \(Tr\) indicates the transpose of a matrix.
where \( v(t) \) is the target’s speed, \( \phi(t) \) is the target’s heading angle (measured counter-clockwise from east), \( a_t(t) \) is the target’s tangential (along-track) acceleration, \( a_n(t) \) is the target’s normal (cross-track) acceleration, and \( t \) denotes time. The standard curvilinear-motion model reduces to three special cases:

\[
\begin{align*}
  a_n = 0, a_t = 0 & \rightarrow \text{rectilinear motion, constant velocity,} \quad (2a) \\
  a_n = 0, a_t \neq 0 & \rightarrow \text{rectilinear motion, constant acceleration,} \quad (2b) \\
  a_n \neq 0, a_t = 0 & \rightarrow \text{circular motion, constant speed}^{1}.
\end{align*}
\]

2.1 Non-Maneuvering Target

A target that does not exhibit tangential or normal accelerations during the tracking process is considered non-maneuvering. Thus, a suitable choice for a motion model is the constant velocity model that stems from (2a).

Evolution of each non-maneuvering target’s state satisfies a discrete-time constant velocity model

\[
x_k = F x_{k-1} + G \nu_{k-1},
\]

where

\[
F = \begin{bmatrix}
1 & 0 & T & 0 \\
0 & 1 & 0 & T \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
G = \begin{bmatrix}
T^2/2 & 0 \\
0 & T^2/2 \\
T & 0 \\
0 & T
\end{bmatrix},
\]

and \( \nu_{k-1} \in \mathbb{R}^2 \) is assumed to be zero mean, normally distributed with covariance \( Q \in \mathbb{R}^{2 \times 2} \), and \( T \) is the sample period.

2.2 Maneuvering Target

Maneuvering target models are naturally turn models, thus, a target that exhibit non-zero, normal (cross-track) accelerations is considered a maneuvering target. Tangential accelerations, (2b) are not considered and, instead, the third case (2c) of the standard curvilinear-motion model is assumed. The case (2c) is known as the coordinated-turn model and it assumes that a target moves with constant speed and constant angular turn-rate \( \Omega = \dot{\phi} \).\textsuperscript{16–18} The continuous-time coordinated-turn model satisfies

\[
\dot{x}(t) = A(\Omega)x(t),
\]

where

\[
A(\Omega) = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & -\Omega \\
0 & 0 & \Omega & 0
\end{bmatrix}.
\]

\textsuperscript{1}Note that velocity is a vector, i.e., \([\dot{\xi} \ \dot{\eta}]^T\), and speed is its magnitude.
2.3 Jump-Markov Target Model

Kinematics of a target that transitions between different motion models is modeled by a jump-Markov system. We make the assumption that between discrete-time instances \( k - 1 \) and \( k \), a target’s state evolves such that it satisfies one of three motion models denoted by \( j \in J = \{1, 2, 3\} \): either constant velocity model \((j = 1)\), a coordinated-turn clockwise model \((j = 2)\), or a coordinated-turn counter-clockwise model \((j = 3)\). The state variable \( r_k = j \) is introduced to denote which motion model the target satisfies. The multitarget density \( p_{\Psi}(X) \) takes the form

\[
x_k = f(x_{k-1}, r_k) + G\nu_{k-1}
= F^{(r_k)}x_{k-1} + G\nu_{k-1},
\]

where \( F^{(1)} \) is defined in (4) and the discrete-time coordinated-turn model in (7) is

\[
F^{(j)} = \begin{bmatrix}
1 & 0 & \frac{\sin(\Omega^{(j)}_{k-1} T)}{\Omega^{(j)}_{k-1}} & \frac{(1-\cos(\Omega^{(j)}_{k-1} T))}{\Omega^{(j)}_{k-1}} \\
0 & 1 & \frac{(1-\cos(\Omega^{(j)}_{k-1} T))}{\Omega^{(j)}_{k-1}} & \frac{\sin(\Omega^{(j)}_{k-1} T)}{\Omega^{(j)}_{k-1}} \\
0 & 0 & \cos(\Omega^{(j)}_{k-1} T) & -\sin(\Omega^{(j)}_{k-1} T) \\
0 & 0 & \sin(\Omega^{(j)}_{k-1} T) & \cos(\Omega^{(j)}_{k-1} T)
\end{bmatrix}, \quad j = 2, 3,
\]

and the turn-rates

\[
\Omega^{(2)}_{k-1} = \frac{a}{\sqrt{\xi^2_{k-1} + \nu^2_{k-1}}}, \quad \Omega^{(3)}_{k-1} = \frac{-a}{\sqrt{\xi^2_{k-1} + \nu^2_{k-1}}},
\]

are estimated at each time-step using the latest target velocity estimates. The parameter \( a \in \mathbb{R} > 0 \) is a constant that represents (cross-track) maneuver acceleration. This method parallels Refs. 9, 13, where (for a sequential Monte-Carlo implementation) each augmented particle with motion model \( i \) at \( k - 1 \) changes motion model at \( k \) based on \( P(r_k = j | r_{k-1} = i) = \pi_{i,j} \) given by an a priori Markov transition matrix \( \Pi \in \mathbb{R}^{|J| \times |J|} \), where \( | \cdot | \) is the cardinality of a set. The three motion models are used as transitional priors in multiple model filters to better reflect the motion of a maneuvering target.

3. BACKGROUND ON PHD

A brief overview on single target estimation and the connection to finite-set statistics and probability hypothesis density is presented in this section. Further details on finite-set statistics and PHD are found in Refs. 19, 20.

The recursive Bayes filter is the primary tool in single sensor, single target tracking. The recursive Bayes filter propagates the belief of the target state through time. However, recursively computing the full posterior belief is computationally intractable for most practical applications. Typical implementations of the recursion (under the assumption of high signal-to-noise ratio) rely on moment matching. That is, the probability (density) of a target’s state can be characterized by the densities first and second-order statistical moments.

The multitarget state \( X = \{x_1, ..., x_V\} \), for \( V \in \mathbb{Z} \geq 0 \), is estimated using finite-set statistics that formulate multitarget problems as random finite sets. A random set \( \Psi_1 \) can be empty \( \emptyset \), containing no target state with probability \( 1 - q \), or contain target state \( x_1 \) with probability \( q \). The union of random sets creates the random finite set \( \Psi = \Psi_1 \cup ... \cup \Psi_V \). The multitarget density \( p_{\Psi}(X) \) takes the form
The probability hypothesis density of a random finite set $\Psi$ on space $S$, 

$$D_\Psi(x) = \int_S \delta_X(x)p_\Psi(X)\delta_X,$$

is analogous to the first-order statistical moment of multitarget distribution $p_\Psi(X)$, where $\delta_X$ denotes the set integral, whereas 

$$\delta_X(x) = \sum_{x \in \Psi} \delta_x,$$

is the set Dirac delta function, and $\delta_x$ is the standard Dirac delta function centered at each $x \in \Psi$. The PHD is defined in the single target state-space and can therefore be thought of as the density of targets at a random vector $x$. The expected cardinality of a random finite set $\Psi$ (the number of targets in $S$) is given by the integral of its PHD over the region $S$,

$$|\Psi| = E[D_\Psi(x)] = \int_S D_\Psi(x) dx. \quad (14)$$

Since the PHD is computationally intractable, it must be approximated through techniques such as Gaussian mixtures or sequential Monte-Carlo methods.

4. MULTITARGET TRACKING AND THE PHD FILTER

This section presents the multitarget filtering problem, the PHD filter, its sequential Monte-Carlo implementation, and the extension of the multiple model PHD filter. Key challenges faced in multitarget filtering at each time-step are: (1) an unknown, varying number of targets, $X = \{x_1, ..., x_V\}$, that appear, persist, or disappear, and (2) a varying number of noisy measurements, $Z = \{z_1, ..., z_M\}$, $M \in \mathbb{Z} \geq 0$, that are obtained by the sensor (including false detections). The multitarget filter has to account for the variation in both the target set $X$ and measurement set $Z$ at each time-step. Due to the randomness in cardinality, the filter recursion should account for probability of survival (the probability that a target will survive from time-step $k-1$ to $k$), probability of detection (the probability that a measurement originates from a target and not a missed detection due to sensor or environment noise), and target birth (a density that can allocate probability (mass) in regions where target appearance is highly likely).
4.1 The PHD Filter

As stated in Section 3, the PHD, \( D_\Psi(x) \), is analogous to the first-order statistical moment of the multitarget probability density function \( p_\Psi(X) \). For filtering notation, we define the PHD of multitarget density \( p_{k|k}(X) \), as \( D_{k|k}(x) \). The PHD filter recursively computes the PHD of \( p_{k|k}(X) \) through prediction and measurement correction steps. Probability of survival and target birth are handled through the PHD predictor\(^{20}\)

\[
D_{k|k-1}(x) = \gamma_{k|k-1}(x) + p_s \int \pi_{k|k-1}(x|x') D_{k-1|k-1}(x')dx',
\]

(15)

where \( \gamma_{k|k-1}(x) \) is the PHD of target births from \( k-1 \) to \( k \), \( \pi_{k|k-1}(x|x') \) is the single target transitional density and \( p_s \) is the probability that a target at \( x' \) at time \( k-1 \) survives to time \( k \). Once a new measurement set \( Z_k \) is obtained, \( D_{k|k-1}(x) \) is updated using the corrector\(^{20}\)

\[
D_{k|k}(x) = \left[ 1 - p_D + \sum_{z \in Z_k} \frac{p_D g_k(z|x)}{K_z(x) + p_D \int g_k(z|x) D_{k|k-1}(x)dx} \right] D_{k|k-1}(x),
\]

(16)

where \( K_z(x) \) is the PHD of clutter (noise), \( p_D \) is the probability of detection, and \( g_k(z|x) \) is the likelihood that a measurement \( z \in Z_k \) originated from a target at \( x \). The right-hand side of (16) contains \( |Z_k| \) additive terms\(^{27}\)

\[
\sum_{z \in Z_k} \frac{p_D g_k(z|x)}{K_z(x) + p_D \int g_k(z|x) D_{k|k-1}(x)dx} D_{k|k-1}(x),
\]

(17)

and the additional terms

\[
(1 - p_D) D_{k|k-1}(x).
\]

(18)

The terms in (17) can be interpreted as the probability that a measurement \( z \in Z_k \) originated from a target whose state is \( x \), and the term in (18) can be interpreted as the probability that a target whose state is \( x \) has not been detected and is therefore in the empty set.

4.2 SMC PHD Filter

A sequential Monte-Carlo (SMC) implementation of the PHD filter partitions the particle set \( \mathcal{P}_{k|k-1} \equiv \{x_{k|k-1}^{(i)}, w_{k|k-1}^{(i)} \}_{i=1, \ldots, N_k} \) to measurements using (17) and (18).\(^{25,27}\) These partitions form particle clusters, \( \mathcal{C}_{k|k-1}(z_c) \), that approximate an additive term of \( D_{k|k-1}(x) \) (17, 18) denoted by \( D_{k|k-1}^{(c)}(x) \) where \( c = 0, 1, \ldots, |Z_k| \). The union of all clusters \( \mathcal{C}_{k|k-1}(z) \) approximates \( D_{k|k-1}(x) \). The SMC PHD filter predictor implements (15) by sampling \( i_b \in 1, \ldots, N_b \cdot |Z_{k-1}| \) particles from birth density, \( x_{k-1}^{(i_b)} \sim b_{k-1}(x|Z_{k-1}) \). The measurement set \( \tilde{Z}_{k-1} \subseteq Z_{k-1} \) is the subset of measurements from the previous measurement set \( Z_{k-1} \) whose particle cluster is empty (a missed detection) and \( N_b \) is the number of particles birthed per missed detection. The birth particle set is appended to the previous surviving particle set with cardinality \( N_{k-1} \), then propagated through the time prediction \( x_{k|k-1}^{(i)} \sim \pi(x_{k|k-1}^{(i)}|x_{k-1}^{(i)}) \), where \( \pi(\cdot|\cdot) \) is the transitional prior given in Section 2.1 (the constant velocity model), and \( i \in 1, \ldots, N_{k-1} + N_b \cdot |\tilde{Z}_{k-1}| \). All corresponding particle weights are set to \( w_{k|k-1}^{(i)} = p_s w_{k|k-1|k-1}^{(i)} \).

The SMC PHD filter analogue of (16) is\(^{27}\)

\[
w_{k|k}^{(i)} = \left[ 1 - p_D + \sum_{z \in Z_k} \frac{p_D g_k(z|x_{k|k-1}^{(i)})}{K_z(x) + p_D \sum_{i=1}^{N_{k-1} + N_b \cdot |\tilde{Z}_{k-1}|} g_k(z|x_{k|k-1}^{(i)}) w_{k|k-1}^{(i)} \right] w_{k|k-1|k-1}^{(i)},
\]

(19)
where the right-hand side of (19) also has $|Z_k| + 1$ additive terms. Particles are partitioned into clusters using the probabilities

$$p_{i,c} = \frac{p_D g_k(z_c | x_{k|k-1}^{(i)})w_{k|k-1}^{(i)}}{K_k(z) + p_D \sum_{l=1}^{N_{k-1} + N_c} g_k(z_c | x_{k|k-1}^{(l)})w_{k|k-1}^{(l)}} ,$$

(20)

and

$$p_{i,0} = (1 - p_D)w_{k|k-1}^{(i)} ,$$

(21)

where $p_{i,c}$ is the probability that measurement $z_c$ is due to a target whose state is $x_{k|k-1}^{(i)}$, and $p_{i,0}$ is the probability that a target whose state is $x_{k|k-1}^{(i)}$ has not been detected and therefore belongs to the empty-set. Partitioning particle $(x_{k|k-1}^{(i)}, w_{k|k-1}^{(i)})$ into cluster $C_{k|k-1}(z_c)$ is done probabilistically through

$$p_i(c) = \frac{p_{i,c}}{\sum_{l=0}^{L} p_{i,l}} ,$$

(22)

where $(x_{k|k-1}^{(i)}, w_{k|k-1}^{(i)})$ is assigned to a cluster $C_{k|k-1}(z_c)$ with probability $p_i(c)$. Particle weights in cluster $C_{k|k-1}(z_c)$ are updated with

$$w_{k}^{(m)} = \frac{p_D g_k(z_c | x_{k|k-1}^{(m)})w_{k|k-1}^{(m)}}{K_k(z) + p_D \sum_{l=1}^{N_{k-1} + N_c} g_k(z_c | x_{k|k-1}^{(l)})w_{k|k-1}^{(l)}} ,$$

(23)

Particles in a cluster $C_{k|k-1}(z_c)$ are resampled according to their weight. All resampled particle weights in cluster $C_{k|k-1}(z_c)$ are set to $p_c/[C_{k|k-1}(z_c)]$, where $p_c = \sum_{m=1}^{[C_{k|k-1}(z_c)]} w_{k}^{(m)}$ is the probability of existence. The probability $p_c$ must be greater than the threshold probability for cluster existence, $p_{ex}$. If $p_c < p_{ex}$, all particles in the cluster are sent to the empty-set where, if a given particle weight is less than the threshold probability for particles in the empty-set $p_{ex}$, they are discarded. Resampled particles are diversified by moving them from $x_k^{(i)} \rightarrow x_k^{(i)*}$ with a Markov-chain Monte-Carlo (MCMC) move step.\(^{13,25,28}\) This diversification step is done through the Metropolis-Hastings update step:\(^{29}\) after resampling, a resampled particle $x_k^{(i)}$ is moved to a new is state $x_k^{(i)*} \sim p(\cdot | x_k^{(i)})$ with probability $\alpha$, where

$$\alpha = \min \left\{ 1, \frac{g(z_k | x_k^{(i)*})p(x_k^{(i)})}{g(z_k | x_k^{(i)})p(x_k^{(i)*})} \right\} .$$

(24)

The proposal density $p(\cdot | x_k^{(i)})$ was chosen as a Gaussian distribution, $p(x_k^{(i)*} | x_k^{(i)}) = p(x_k^{(i)*}) = N(\hat{x}_k^{(i)*}; \mu, \epsilon \hat{C})$, where, $\hat{C}$ is the cluster’s sample covariance, and $\epsilon$ was chosen such that the cluster’s underlying distribution was in the support of the proposal.

### 4.3 Multiple Model PHD Filter

The multiple model probability hypothesis density (MMPHD) filter extends the single target multiple model particle filter described in Ref.\(^{9}\). Particles are propagated through the PHD predictor using the Markov transition probability $\pi(x | x')$, i.e., the component of (15) pertaining to persisting particles. The transition probability $\pi(x | x')$ is usually the transitional prior given in Section 2.1. To account for maneuvering targets, the transitional prior is treated as a jump-Markov system such that $\pi(\cdot | \cdot)$ propagates particles in a way that can reflect a maneuvering target’s motion. Thus, each particle, $x_{k|k-1}^{(i)}$, is augmented with an indicator, $r_k$, specifying which motion model $j \in J$ the particle is propagated with in the prediction step, $x_{k|k-1}^{(i)} \sim \pi(x_k | x_{k|k-1}^{(i)}, r_k)$, where
\( \pi(\cdot|\cdot) \) in the multiple model context represent the transitional priors detailed in Section 2.2. All other aspects of the MMPHD stay the same as in the PHD filter described in Section 4.2. That is, the probability of survival, probability of detection, target birth, and likelihood are independent of \( r_k \) which implies that

\[
g_k(z|x^{(i)}_{k|k-1}) = g_k(z|x^{(i)}_k) = g_k(z|x^{(i)}_k). \tag{25}
\]

After resampling, the model \( r^{(i)}_k \) for each surviving augmented particle changes model with probability \( P(r^{(i)}_k = j| r^{(i)}_{k-1}) \). Therefore, as the number of resampled particles with model \( r_k = j \) increases over the other models, so does the probability \( P(r_k = j| z_k \in Z_k) \).

5. SIMULATION

The PHD and MMPHD filters are implemented using real-time bearing data of a support ship from an UUV fitted with a hydrophone array. Analysis of filter performance is done off-line via 100 Monte-Carlo (MC) runs per scenario. Global positioning system (GPS) data of the support ship and inertial position estimates of the UUV gathered throughout the sea-trial is used to evaluate the performance of the PHD and MMPHD filters. Root-mean square position error

\[
RMSE_k = \sqrt{\frac{1}{M} \sum_{m=1}^{M} \left( \xi_k^m - \xi_k \right)^2 + \left( \eta_k^m - \eta_k \right)^2}. \tag{26}
\]

is used as the metric of performance where \( M \) is the number of MC runs, \( (\xi_k^m, \eta_k^m) \) is the estimated position at time-step \( k \) for MC run \( m \) and \( (\xi_k, \eta_k) \) are the true target position,

5.1 Filter Initialization

The filter is initialized by using the first measurement set \( Z_0 \). The target birth density in target position is uniformly distributed in an area that resembles a sector of a circle centered on the UUV defined by the sensor’s maximum (or maximum effective range) \( r_{\text{max}} \) and the standard deviation of the sensor \( \sigma_\theta \). The target birth density in velocity is a uniform distribution around the maximum allowed target speed (a user-defined parameter) \( v_{\text{max}} \). Thus, a birth sample, \( x^{(i_s)} \) is distributed

\[
b_{k-1}(x|z) \equiv [\hat{r} \cos (z \in Z_0 + \sigma_\theta), \hat{r} \sin (z \in Z_0 + \sigma_\theta), v_x, v_y]^T, \tag{27}
\]

where \( \hat{r} \sim \mathcal{U}[0, r_{\text{max}}] \), \( \sigma_\theta \sim \mathcal{U}[-3\sigma_\theta, 3\sigma_\theta] \), \( v_x \sim \mathcal{U}[-v_{\text{max}}, v_{\text{max}}] \), and \( v_y \sim \mathcal{U}[-v_{\text{max}}, v_{\text{max}}] \).

5.2 Measurement Ambiguity

The hydrophone array output is with respect to the UUV body. Zero degrees corresponds to the bow of the UUV, and 180 degrees corresponds to the tail or the stern of the UUV. A beamformer algorithm on-board the UUV measures the bearing to a source of noise from acoustic measurements acquired by the hydrophone array. The measurements that drive the PHD filter are from the output of the beamformer. Due to port/starboard ambiguity, the beamformer output produces two measurements per target. Certain UUV maneuvers can help eliminate port/starboard ambiguity. That is, for certain maneuvers, the likelihood of the incorrect track decreases.

After tracks corresponding to port and starboard bearing measurements have become distinguishable, use of Bayes factors in target trajectory estimation can continue to resolve port/starboard ambiguity without the necessity of specific UUV maneuvers. Bayes factors quantify the support of a hypothesis used to characterize data with respect to other hypotheses. At time-step \( k \), assuming particles for each track are normally distributed, port/starboard hypotheses are constructed using the port and starboard target state estimates and empirical sample covariances \( (\hat{x}^k_p, C^k_p) \), and \( (\hat{x}^k_s, C^k_s) \). The previous best estimate \( \hat{x}_{k-1} \) is the datum that port/starboard hypotheses characterize. The Bayes factor is
\[ BF(x) = \frac{P(H_P|D)P(H_S)}{P(H_S|D)P(H_P)} \]
\[ = \frac{P(\tilde{x}^k_P, \tilde{C}^k_P|x_{k-1})g(z_S|\tilde{x}^k_S)}{P(\tilde{x}^k_S, \tilde{C}^k_S|x_{k-1})} \]
\[ = \frac{P(\tilde{x}^k_P, \tilde{C}^k_P|x_{k-1})\sum_{w \in \tilde{x}^k_P} w_S}{P(\tilde{x}^k_S, \tilde{C}^k_S|x_{k-1})\sum_{w \in \tilde{x}^k_S} w_P} \]  

(28)

where \( w_P \) and \( w_S \) are the weights corresponding to particles in \( \tilde{x}^k_P \) and \( \tilde{x}^k_S \), respectively. Thus, at time-step \( k \), the correct target track will have more support. That is, a comparison between \( BF(x) \) and \( BF^{-1}(x) \) is used to accept \( \tilde{x}^k_P \) or \( \tilde{x}^k_S \) as the most likely target position.

5.3 Filter Parameters

Excluding outliers, measurement noise is approximately normally distributed with mean, \( \mu_\theta = 1.29^\circ \), and standard deviation, \( \sigma_\theta = 1.39^\circ \). From data analysis, measurements tend to saturate approximately between \((0^\circ, 30^\circ)\), and \((150^\circ, 180^\circ)\). To address measurement saturation in the measurement likelihood function, measurement noise standard deviation \( \sigma_\theta \) is rewritten to explicitly display dependence on \( z \),

\[ \sigma_\theta(z) = \begin{cases} 
1.39^\circ & \text{if } 30^\circ < z < 150^\circ \\
30^\circ & \text{if } 0^\circ \leq z \leq 30^\circ \text{ or } 150^\circ \leq z \leq 180^\circ 
\end{cases} \]  

(29)

A total of \( N = 5000 \) particles where used per track as well as \( N_b = 5000 \) particles used for each \( z \in \tilde{Z}_{k-1} \). Unless otherwise noted, all filters use the parameters: \( p_D = 0.99 \), \( p_s = 0.75 \), \( K = \lambda/2\pi \), \( \lambda = 1 \), \( p_{ce} = 0.05 \) and \( p_{es} = p_s/2N \).

5.4 Stationary target

The first scenario shows the maneuvering UUV tracking the mostly stationary support ship for 26 minutes, as shown in Fig. 1. At the start, the ship and UUV are 340 m apart. The relative bearing estimate produced by the on-board beamformer is shown in Fig. 2. The two horizontal lines in Fig. 2 denote where relative bearing estimates begin to saturate. Fig. 3 shows the difference, \( \Delta \theta \), between the true (given by the four quadrant arctangent based on GPS and inertial estimate data) and best-case bearing estimate. The best-case bearing estimate is defined as the measurement, port or starboard, that is closest to the true bearing. The root-mean square position error results of the 100 Monte-Carlo runs is shown in Fig. 4.

For the first two minutes, the filter estimates the ship’s location and maintains port/starboard certainty. A change in the UUV’s heading is executed at about 2 minutes after the start of the mission when small errors in bearing estimates (roughly 5°) affect the estimation error. After the bearing estimate settles, the filter again converges. At approximately 4.5 minutes, a change in UUV heading is commanded, leading to measurement saturation for approximately 30 seconds. This leads to the jump in bearing error seen between 4.5 and approximately 5 minutes in Fig. 3. The jump is caused by the best-case bearing estimate switching from the starboard measurement to the port measurement.

Approximately from 6 to 12 minutes into the track, rapidly, repeated semi-informative births coupled with large perturbations in bearing estimate, including two additional instances where bearing estimates saturate cause the filter performance to degrade. This is because the filter goes through a cycle of discarding particles and spawning new ones. Because birth particles are sampled using Equation (27), tracks have limited information on the target state, thus taking longer to converge.

Approximately 12 minutes after the start of the mission, the track begins to again converge with the exception of a few UUV heading changes that cause the bearing estimates to saturate. The sharp increase in error from the 16 to 24 minute mark is due to the exchange of impoverished particles pertaining to the incorrect track.
Though impoverished, the probability of cluster existence is still above the threshold, and the particles are not immediately sent to the empty-set. After a few iterations however, the probability of existence falls below the threshold, and particles are moved to the empty-set where they persist to the next time-step or are discarded. Because the measurement corresponding to the ship’s true position has no particles attributed to it, a new set of particles are birthed and the estimate continues to converge.

5.5 Maneuvering Target 1

The second scenario shows the maneuvering ship following a north/south trajectory and the UUV following a mostly east/west trajectory for approximately 25 minutes, as shown in Fig. 5. At the start of the mission, the ship and UUV are 898 m apart. Beamformer data and best-case bearing estimate error are shown in Figs. 6 and 7, respectively.

During some portions of this scenario the UUV was performing safety behaviors to return to the center of the predefined region of operation rather actively tracking the surface ship. In such cases the UUV’s trajectory is not optimal in the context of information gain and observability of the ship’s range. A UUV trajectory heading that increases information and observability of the ship would be parallel to the smallest eigenvector of the track empirical covariance. During the safety maneuvers, the UUV travels, for the most part, perpendicular to the smallest eigenvector of the track empirical covariance. Additionally, most measurements are near or in regions where bearing estimates saturate. Closer inspection of the ship’s trajectory shows a variety of maneuvers performed at different turn-rates. Thus, the PHD and MMPHD filters are expected to perform poorly. Results of the traditional implementation of the SMC PHD filter and the proposed SMC MMPHD filter are shown in Fig. 8. The typical acceleration maneuver is set to \( a = 0.1 \, m/s^2 \) and the Markov transition matrix for the multiple model PHD is

\[
\Pi = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}. \tag{30}
\]
For the first 2 minutes of the mission, estimates begin to converge. The MMPHD estimate shows a faster convergence rate than that of the PHD. Three minutes into the mission, a change in the UUV’s heading cause measurements to saturate. Both filters retain the track, however, the MMPHD maintains the particle cloud closer to the ship’s true position. This is because the constant velocity model in the PHD performs poorly due to the noise in the ship’s trajectory. The particles in the MMPHD are (by construction) better suited for the noise in the ship’s trajectory. The ship maneuvers approximately 3.5 minutes into the track, causing both the PHD and MMPHD estimates to diverge. The MMPHD maintains a lower estimate error throughout the maneuver. Another slight maneuver occurs at approximately 4.5 minutes. The maneuver increases the estimate error in both filters. The MMPHD error remains lower than that of the PHD.

The ship performs a sharp maneuver at approximately 6 minutes. Throughout the maneuver, bearing measurements are mostly constant, causing the ship’s range to become unobservable. The mostly constant bearing measurements causes particles in both filters to diverge from the true state of the ship, leading to the particles moving to the empty-set, while new particles are generated. The maneuvers performed by the UUV do not help eliminate port/starboard ambiguity, thus, both filters estimate the incorrect track. Despite another ship maneuver 8 minutes into the mission, the filters maintain the estimate on the incorrect track. The UUV maneuver at approximately 10 minutes into the mission cause measurements to saturate. The particles weights in the PHD filter fall below the threshold of existence, birthing a new set of particles. Both filters begin to converge on the true ship position, with the MMPHD converging at a faster rate.

Measurements saturate around 13 minutes, causing the PHD filter to discard the current particle set and birth a new set. At 14 minutes, the ship performs another sharp maneuver. The sharp maneuver, coupled with large errors in the bearing estimate, and measurement saturation between 13.5 and 20.5 minutes are responsible for the subsequent divergence in track estimate. Once measurement errors and saturation subside, both filters are shown to converge at approximately the same rate.
5.6 Maneuvering Target 2

The last scenario shows the ship following a box-pattern trajectory centered around the UUV. The initial range to the ship is 893 m and tracking lasts for approximately 13 minutes. The trajectories of both, the ship and the UUV are shown in Fig. 9. Beamformer data and best-case bearing estimate error is shown in Figs. 10, 11, respectively. Due to measurement saturation being more prevalent in this scenario, the probability of detection is set to $p_D = 0.8$. The root-mean square position error results are shown in Fig. 12. From analysis of the ship’s trajectory throughout the tracking process, the typical acceleration maneuver is set to $a = 0.2 \text{ m/s}^2$. The Markov transition matrix for the multiple model PHD filter is

$$
\Pi = \begin{bmatrix}
0.65 & 0.175 & 0.175 \\
0.25 & 0.65 & 0.1 \\
0.25 & 0.1 & 0.65
\end{bmatrix}.
$$

The data in Fig. 12 shows that the SMC MMPHD filter almost always outperforms the traditional SMC PHD filter. The first maneuver begins approximately three minutes after the start of the mission. The estimation of the MMPHD is lower than that of the traditional PHD, and the MMPHD also shows faster estimate convergence.

The second maneuver takes place approximately 6.33 minutes after the start of the mission. This maneuver takes place while the ship is directly behind the array (end-fire) where the array cannot generate useful bearing measurements. Since the estimated track error is already increasing at this time, both track estimates diverge. In both filters, port/starboard ambiguity leads to establishing the incorrect track as the most likely. The correct track is regained once the ship is out of end-fire. This is because the particle cloud pertaining to the true target measurement becomes more likely, that is, the Bayes factor corresponding to the track shows more support over the other track.

The final maneuver occurs approximately 9.83 minutes into the track, during which the bearing measurement is nearly constant, causing the ship’s range to be unobservable. The filter begins to converge soon after, but saturated measurements that appear at 12 minutes cause the estimate to diverge. After the UUV maneuvers such that the ship is located broadside of the array, the filter converges to the true ship position.

6. CONCLUSION

A multiple model extension to the PHD filter is presented. The performance of a SMC PHD and the proposed SMC MMPHD is evaluated using Monte Carlo simulations on a real data set of a unmanned underwater vehicle performing bearings-only tracking at-sea. Though the MMPHD has been shown to almost always outperform the SMC PHD, the MMPHD requires that parameters such as the Markov transition matrix and typical acceleration must be known a priori, which hinders the performance of the filter for rapidly maneuvering targets and targets that perform maneuvers at various turn-rates. However, ease of implementation make it suitable for applications with a priori knowledge of target behavior such as the maneuver frequency and turn-rate.
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