WLAN and WPAN Coexistence in UL Band

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Abstract—Wireless local- and personal-area networks provide complimentary services in the same unlicensed (UL) radio frequency band of operation. As the mutual benefits of utilizing these services become increasingly apparent, the likelihood of mutual interference may also increase. A method was developed for examining wireless services coexistence in order to evaluate the impact that interference may have on network performance. The methodology for the analysis was centered on deriving a closed-form solution for the probability of collision $Pr[C]$ in terms of the network and radio propagation parameters. In addition, a set of measures of performance was derived based on $Pr[C]$. In this fashion, the network performance was investigated in regards to the presence of interference. The approach was then illustrated by examining the coexistence between 802.11b and Bluetooth UL band wireless services and summarizing the impact on network performance.

Index Terms—Bluetooth, coexistence, IEEE 802.11, wireless local area network, wireless personal area network.

I. INTRODUCTION

As unlicensed (UL) band utilization for daily office functions increases, it becomes important to understand how different wireless services, operating in the same band, may affect each other. As the wireless applications move from ones of convenience to ones of reliance, the capability to predict potential interoperability issues is imperative in order to forestall negative consumer opinion. This issue has become even more pressing with the advent of the Bluetooth (BT) standard [1], [2]. BT-compliant devices developed at or near their target price should facilitate the rapid introduction of new services. However, the issues surrounding their coexistence need to be addressed before the interoperability problems, whether speculative or actual, become a deterrent to their commercial acceptance.

The goal of the research presented in this paper was to derive a closed-form analytical solution that could be used to evaluate the coexistence issue under many network scenario conditions. A methodology for examining coexistence when there was uncertainty in the expected network traffic activity or uncertainty in the radio environment for the installation’s operation was also an important consideration. To illustrate the approach, the coexistence between 802.11b and BT piconets was addressed, and specifically, the impact of BT piconets on 802.11b station when they are collocated in the same region was analyzed.

Coexistence analysis between the 802.11b and BT piconet has been addressed in [3]–[8]. In this paper, a more general analytical model is presented, based on using a log-normal shadowing radio propagation model and based on using an adjacent band interference model. The derivation for the coexistence model was divided into two parts. First, the measures of performance for the network are derived in terms of the probability of collision between the BT piconet interference signal and the desired 802.11b signal in Section II. Next, the coexistence model is derived in Section III. Analysis based on the resulting closed-form solution is presented in Section IV, with conclusions given in Section V.

II. MEASURES OF PERFORMANCE DERIVED IN TERMS OF PROBABILITY OF COLLISION

Network performance can be evaluated from a number of viewpoints using various measures of performance (MOPs). The relevance of each MOP is dependent on the specific network requirements. In this section, three network MOPs are considered for evaluating the impact of BT on the 802.11b network performance: packet error rate (PER), number of packet retransmissions (RT), and transmission latency (S). An expression for each MOP is first derived in terms of probability of collision $Pr[C]$. A collision $C$ defines the event where one or more BT signals corrupt an 802.11b packet, such that retransmission of the packet is required. The derivation of $Pr[C]$ is presented in Section III.

By deriving the MOPs in terms of $Pr[C]$, the range over $Pr[C]$ where the performance specifications for the $i$th MOP is achieved or not achieved can be determined. That is

$$V_i \equiv \{Pr[C] \mid \text{where } i\text{th MOP is not satisfied}\}$$

$$U_i \equiv \{Pr[C] \mid \text{where } i\text{th MOP is satisfied}\}. \quad (1)$$

Then, combining the results for the individual MOPs

$$V = \bigcup_i V_i \quad \text{and} \quad U = \bigcap_i U_i \quad (2)$$

the overall performance criteria are obtained. The objective is to determine the combination of BT network parameters and radio propagation environments, resulting in

1) $Pr[C] \in V \Rightarrow 802.11b$ network performance is likely to be impaired by BT;
2) $Pr[C] \in U \Rightarrow 802.11b$ network performance is not likely to be impaired by BT.
A. MOP-Packet Error Rate

Given \( N \), 802.11b packets are transmitted. Assuming the packet collisions with BT are independent and identically distributed (i.i.d.), then the probability that PER exceeds a PER threshold \( \gamma_{\text{PER}} \) is a binomial distribution [9]

\[
\Pr[\text{PER} > \gamma_{\text{PER}}] = 1 - \Pr[\text{PER} \leq \gamma_{\text{PER}}]
\]

\[
\Pr[\text{PER} \leq \gamma_{\text{PER}}] = \sum_{n=0}^{N} \binom{N}{n} (\Pr[C])^n \times (1 - \Pr[C])^{N-n}
\]

where, for ease of notation, \( \gamma_{\text{PER}} \in [0/N, 1/N, \ldots, N/N] \). A Gaussian approximation to the binomial distribution [10] can be used to estimate (3), given that \( N \) is sufficiently large, such that \( N \times \Pr[C] \gg 1 \)

\[
\Pr[\text{PER} > \gamma_{\text{PER}}] 
\approx \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{N (\gamma_{\text{PER}} - \Pr[C])}{\sqrt{2} \gamma_{\text{PER}} \Pr[C] (1 - \Pr[C])} \right)
\]

where \( \text{erf}(\cdot) \) is the standard error function. In Fig. 1(a), graphs of equal probability for \( \Pr[\text{PER} > \gamma_{\text{PER}}] \) are depicted based on (4).

A common network performance specification is based on the expected PER \( E[\text{PER}] \). From both (3) and (4), it is straightforward to obtain \( E[\text{PER}] = \gamma_{\text{PER}} = \Pr[C] \). For the 802.11b, \( E[\text{PER}] \) is specified to be less than or equal to 8%. This value was used to specify an upper bound on \( \Pr[C] \) such that no or little impairment of the network was assumed when \( E[\text{PER}] \leq 0.08 \), i.e., \( U_{\text{PER}} = \{ \Pr[C] \mid E[\text{PER}] \leq 0.08 \} = \{ \Pr[C] \mid \Pr[C] \leq 0.08 \} \). The network was assumed significantly impaired if the average PER exceeded 20%, \( V_{\text{PER}} = \{ \Pr[C] \mid \Pr[C] \geq 0.2 \} \). For the purpose of the study presented in this paper, the overall performance criteria[2]) was based solely on PER, \( U = U_{\text{PER}} \) and \( V = V_{\text{PER}} \).

B. MOP Number of Packet Retransmissions

The number of packet retransmissions \( RT \) required on average to successfully transmit an 802.11b packet from the 802.11b access point (AP) to the 802.11b station (STA) provides a measure of the latency associated with network impairment. Under the assumption that the packet collisions are i.i.d., then the probability that \( RT \) is less than an RT threshold \( \gamma_{RT} \) is a geometric distribution [9]

\[
\Pr[R_T < \gamma_{RT}] = \sum_{r=0}^{\gamma_{RT}} (\Pr[C])^r (1 - \Pr[C])^r
\]

\[
= 1 - \Pr[C]^{\gamma_{RT}}.
\]

Using properties of geometric series, it is straightforward to show the mean \( E[RT] \) and standard deviation \( \sigma_{RT} \) are

\[
E[RT] = \frac{\Pr[C]}{1 - \Pr[C]}
\]

\[
\sigma_{RT} = \sqrt{\frac{\Pr[C]}{1 - \Pr[C]}}.
\]

\( E[RT] \) and \( E[RT] + \sigma_{RT} \) are graphed in Fig. 1(b).
C. MOP Transmission Latency

Transmission latency is a measure of the expected time for a packet to be successfully received. For the 802.11b, the normal time required for successful packet transmission and reception \( T_{\text{normal}} \) is composed of the time for detecting medium activity, the time for packet (including header) transmission, and the time for the receiver to acknowledge packet reception. When an acknowledgment is not received by the Tx or when an incorrect checksum is received, then the Tx retransmits the same packet. When retransmission is required, an additional overhead time involving a random backoff time is required to assist in resolving medium contention between 802.11b transmitters [11]. The expected retransmission overhead \( T_{\text{RT}} \) was modeled in terms of \( T_{\text{normal}} \). 

\[ T_{\text{RT}} = \alpha T_{\text{normal}}, \]  

where \( \alpha \) is a proportionality constant. This formulation facilitates the evaluation of the expected transmission latency \( E[S] \). The time duration for transmitting a packet when \( k \) packet collisions have occurred is, therefore

\[ T_{\text{normal}} + k(T_{\text{normal}} + T_{\text{RT}}) = T_{\text{normal}}(1 + k(1 + \alpha)). \]  

(7)

Combining (7) with the geometric distribution for the probability of \( k \) packet collisions occurring prior to receiving a valid packet, \( E[S] \) is obtained by

\[ E[S] = T_{\text{normal}} \left[ 1 + \sum_{k=1}^{\infty} (1 + \alpha) k (\Pr[C])^k (1 - \Pr[C]) \right]. \]  

(8)

Using the convergence property of the infinite series, \( \sum_{n=1}^{\infty} n^{\beta/\alpha} = \beta/(1 - \beta)^2 \), the normalized transmission latency is

\[ \frac{E[S]}{T_{\text{normal}}} = \frac{1 + \alpha \Pr[C]}{1 - \Pr[C]}, \]  

(9)

Fig. 1(c) depicts normalized transmission latency versus \( \Pr[C] \) for \( 0 < \alpha \leq 0.1 \). As can be seen by examining both the graph and (9), the impact of \( T_{\text{RT}} \) on the transmission latency is negligible when \( \alpha \times \Pr[C] \ll 1 \). The 802.11b protocol has provisions for fragmenting long packets in order to decrease the collision probability at the expense of higher overhead. The work presented in this paper does not consider packet fragmentation and focuses solely on fixed-length packets, specifically 1500 bytes.

III. COEXISTENCE MODEL

A. Expected Number of Interferers

In this section, the expected number of BT piconets \( N_{\text{BT}} \) having sufficient power to cause interference to an 802.11b STA is determined. The analysis was based on first examining the relative received powers at the STA from the desired signal source, 802.11b AP, and BT piconets within radius \( D \) of the STA. The geometry and corresponding parameters used in the analysis are illustrated in Fig. 2. The BT piconets were assumed to be uniformly distributed about the STA. Given the BT piconet density of \( D_{\text{BT}} \) piconets/m², then the expected number of BT piconets is

\[ N_{\text{BT}}(\Gamma) = A_{\text{eff}}(\Gamma, d_s|D) D_{\text{BT}} \]  

(10)

where \( A_{\text{eff}}(\Gamma, d_s|D) \) is the effective interference area given radius \( D \). \( A_{\text{eff}}(\cdot) \) estimates the area within a circle centered at the STA with radius \( D \), where the interference signal from the BT piconets exceeds the normalized interference to signal power ratio threshold \( \Gamma \). \( A_{\text{eff}}(\cdot) \) is also dependent on the distance between the AP and the STA \( d_s \), where the dependency is governed by the radio propagation path-loss characteristics. The normalized interference to signal power ratio threshold is given by

\[ \Gamma = \gamma_{I/S} - \Omega_{\text{BT}} + \Omega_{\text{AP}} \, (\text{dB}) \]  

(11)

where \( \Omega_{\text{BT}} \) and \( \Omega_{\text{AP}} \) are the transmit powers in dBm for BT and 802.11b, respectively, and \( \gamma_{I/S} \) is the interference to signal power threshold in dB, i.e., the threshold at which the interference signal corrupts the 802.11b transmission.

The effective interference area was determined using an approach similar to Jake’s method [12] for determining the percentage of the useful coverage area within a cell’s boundary when taking into account the effects of shadowing. That is

\[ A_{\text{eff}}(\gamma_{I/S} , d_s | D) = \int_0^{2\pi} \int_0^D \Pr \left( \frac{\Omega_I(r)}{\Omega_S(d_s)} > \gamma_{I/S} \right) r \, dr \, d\theta \]  

(12)

where \( \Pr(\Omega_I(r)/\Omega_S(d_s) > \gamma_{I/S}) \) is the probability that the interference power \( \Omega_I(r) \) at radius \( r \) exceeds the received signal power from the STA \( \Omega_S(d_s) \) by \( \gamma_{I/S} \). Both the signal power and interference power were based on a log-normal shadowing model [13]

\begin{align*}
\Omega_I(r) &= \Omega_{\text{BT}} - \left[ P_L(d_0) + 10n \log_{10} \left( \frac{r}{d_0} \right) + X_I \right] \, (\text{dBm}) \\
\Omega_S(d_s) &= \Omega_{\text{AP}} - \left[ P_L(d_0) + 10n \log_{10} \left( \frac{d_s}{d_0} \right) + X_S \right] \, (\text{dBm}) 
\end{align*}

(13)

where

- \( P_L(d_0) \) path loss at reference distance \( d_0 \);
- \( n \) path-loss exponent;
\(X_I\) and \(X_S\) are zero-mean log-normal distributed random variables (RVs) with standard deviations \(\sigma_I\) and \(\sigma_S\), respectively.

For \(\Omega_S(d_S)\), (13) is the standard large-scale propagation model based on log-normal shadowing with RV \(X_S\) modeling the variations in received signal power due to variations in coverage area obstructions. For \(\Omega_T(r)\), the distance \(r\) represents the distance between the STA and the centroid of the nodes in the BT piconet. The RV \(X_I\) is used to model both the effects of shadowing and the variations in signal power due to the variation in the BT nodes’ location about their centroid. Using (13) and assuming \(X_I\) and \(X_S\) are independent RVs, then the interference-to-signal ratio is shown in (14) at the bottom of the page, where \(X_I/S\) is a zero-mean log-normal distributed RV with standard deviation \(\sigma_{I/S} = \sqrt{\sigma_I^2 + \sigma_S^2}\).

Therefore, \(\Omega_{I/S}(r; d_S)\) is a log-normal distributed RV with mean

\[
\Omega_{I/S}(r; d_S) = \Omega_{BT} - \Omega_{AP} - 10n \log_{10} \left( \frac{r}{d_S} \right) \text{ (dB)} \tag{15}
\]

and standard deviation \(\sigma_{I/S}\). Using this formulation for \(\Omega_{I/S}(r; d_S)\), (12) can be solved in a similar manner as the percentage coverage area as formulated in [14]

\[
A_{\text{eff}}(\Gamma, d_S) = \pi D^2 \left[ 1 - \text{erf} \left( \frac{a}{\sqrt{2}} \right) \right] \times \left[ 1 - \text{erf} \left( \frac{1 - ab}{b} \right) \right] \tag{16}
\]

where \(a = \left( \gamma_{I/S} - \Omega_{I/S}(r; d_S) \right) / \sqrt{2} \sigma_{I/S} = \Gamma + 10n \log_{10}(r/d_S) \) and \(b = (10n \log_{10}(e) / \sqrt{2} \sigma_{I/S})\).

In (16), the effective interference area is based on restricting the BT interferers to those within a radius \(D\) of the STA. Since a BT piconet could cause interference as long as \(\Omega_I(r)/\Omega_S(d_S) > \gamma_{I/S}\), regardless of its distance to the STA, (16) is evaluated as \(D \to \infty\)

\[
A_{\text{eff}}(\Gamma, d_S) = \lim_{D \to \infty} A_{\text{eff}}(\Gamma, d_S) \mid D = \pi (d_S)^2 \exp \left[ \frac{2 \left( \gamma_{I/S}^2 - 10n \log_{10} \left( e \right) \right)}{\left( 10n \log_{10} \left( e \right) \right)^2} \right] \tag{17}
\]

The normalized effective interference area \(A_{\text{eff}}(\Gamma, d_S) / \pi d_S^2\) is graphed in Fig. 3 for typical parameter ranges for the 802.11b and BT services. The range for \(\Gamma\) is based on typical BT Tx power of 0 dBm. BT Tx power can be extended to 20 dBm to achieve an increased coverage range, given that power control is employed. Typical 802.11b Tx power is 20 dBm. The minimum value for \(\gamma_{I/S}\) is based on an empirical study reported in [5] and [8]. Based on the study’s results, the 802.11b can provide reliable service when \(\gamma_{I/S} < -10\) dB is maintained. The maximum value for \(\gamma_{I/S}\) takes into account the following two points: BT acts as a narrow-band jammer to the 802.11b DSSS signal and \(\gamma_{I/S}\) is dependent on the frequency offset between the BT and 802.11b carrier frequencies \(f_{\text{offset}}\). As \(f_{\text{offset}} > W_S/2\), where \(W_S\) is the bandwidth of the 802.11b Tx signal, the value of \(\gamma_{I/S}\) required to cause interference increases due to the effects of DSSS demodulation on a narrow-band interference signal. Therefore, the upper value for \(\Gamma\), in Fig. 3, was based on determining the value of \(\Gamma\) such that \(A_{\text{eff}}(\Gamma, d_S) \approx 1\) m^2 given \(d_S = 20\) m. The radio propagation parameters were based on typical measured values for a single office building floor [14] with \(2 < n < 4\) and log-normal shadowing standard deviation between 3–9 dB typical \(\Rightarrow 5 < \sigma_{I/S} < 11\) dB. This assumes that the BT nodes for a given piconet are in close proximity to each other. Therefore, the variation of the received interference power, due to the variation in the BT nodes’ location about their centroid, is minimal. Fig. 4 contains graphs for normalized \(N_{\text{BT}}(\Gamma)\), \(N_{\text{BT}}(\Gamma) / D_{\text{BT}} = A_{\text{eff}}(\Gamma, d_S)\), for \(\Gamma = 10\) dB (i.e., \(\Omega_{\text{BT}} = 0\) dBm, \(\Omega_{\text{AP}} = 20\) dBm, \(\gamma_{I/S}(0) = -10\) dB) and, for a typical 802.11b operational range, \(0 < d_S < 20\) m.

### B. Probability of Time Coincidence

BT and 802.11b relative timing is illustrated in Fig. 5. The analysis presented in this section is an extension of the model developed in [4]–[6] for the probability of time coincidence. Consistent with [4]–[6], the BT piconet was assumed to be using single time slot packets only, as shown in the figure. The single time slot results in the highest BT interference [4]–[6] since it increases the likelihood that BT will be time coincident with the 802.11b packet transmission. Under this assumption, a single BT packet is transmitted at each frequency hop with the BT time slot duration, \(T_{\text{BT}} = 0.25\) ms, and the transmission time within the time slot, \(\tau_{\text{BT}} = 300\) μs. The 802.11b packet duration \(T_P\) can be up to 1210 μs for a high-rate packet transmis-
Fig. 4. Normalized number of BT interferers exceeding interference threshold based on distance between 802.11 AP and STA.

Fig. 5. Relative timing between BT Tx time slots and 802.11 packet.

The relative timing offset between the BT packet sequence and 802.11b packet transmission is modeled as a uniform RV $T_{\text{offset}} \in [0, T_{\text{BT}}]$. Under this assumption, the number of BT time slot transmissions overlapping in time with the 802.11b packet transmission is either $n_\tau$ or $n_\tau - 1$. It is straightforward to show that

$$n_\tau = \left\lceil \frac{T_P + n_{\text{BT}}}{T_{\text{BT}}} \right\rceil$$

(18)

where $\lceil \cdot \rceil$ is the ceiling function. The value of $n_\tau$ represents the maximum number of BT time slots that the 802.11b packet overlaps in time. The corresponding probabilities $Pr[n_\tau]$ and $Pr[n_\tau - 1]$ were obtained by observing that there are two mutually exclusive events that lead to $n_\tau$. The distinction between the two events is based on setting $T_{\text{offset}} = \gamma_{\text{BT}}$ and observing whether or not the 802.11b packet simultaneously overlaps the first and last of the $n_\tau$ BT time slots. If simultaneous overlap occurs, then the $T_{\text{offset}}$ range corresponding to the event $n_\tau$ is $(n_\tau - 1)T_{\text{BT}} - T_P < T_{\text{offset}} \leq n_{\text{BT}}$. If simultaneous overlap does not occur, then the $T_{\text{offset}}$ range corresponding to the event

$$n_\tau \text{ is } \{0 \leq T_{\text{offset}} < \gamma_{\text{BT}}\} \cup \{n_\tau T_{\text{BT}} - T_P < T_{\text{offset}} \leq \gamma_{\text{BT}}\}.$$ 

Combining the ranges for $T_{\text{offset}}$ with $T_{\text{offset}}$’s distribution gives

$$Pr[n_\tau] = \frac{T_{\text{BT}} + T_P - (n_\tau - 1) T_{\text{BT}}}{T_{\text{BT}}}$$

$$Pr[n_\tau - 1] = 1 - Pr[n_\tau].$$

(19)

In the analysis, it was assumed that if any portion of the 802.11b packet was time coincident with a BT transmission, then 802.11b packet retransmission is required, given frequency coincidence as discussed in Section III-C. This is justified since $\gamma_{\text{BT}}$ is much larger than the 802.11b symbol period.

C. Probability of Frequency Coincidence

The collision probability is also dependent on the probability the signals from the two services are frequency coincident. Frequency coincidence occurs when the transmit frequencies of each service result in an 802.11b packet error, assuming the two signals are time coincident. The BT Tx bandwidth is approximately 1 MHz, and, due to the frequency hopping schedule, uniformly distributed over the UL bandwidth, $B_{\text{BT}} = 80$ MHz. The 802.11b DSSS Tx bandwidth is nominally $B_{\text{S}} = 20$ MHz. Therefore, a first-order approximation to the probability of frequency coincidence $Pr[C_f]$ is based on the likelihood that the BT 1 MHz Tx bandwidth occurs within the 20-MHz 802.11b Tx bandwidth, i.e., $Pr[C_f] = 20/80$. This is the method used in [4]–[6]. The limitation to this approach is that it does not take into account adjacent channel interference. The method outlined in this section and used in evaluating the coexistence in Section IV does incorporate an out-of-band interference model.

Based on empirical data [8], the 802.11b can provide reliable service in the presence of narrow-band interference occurring within the passband, given $\gamma_{1/5}(0) < -10$ dB. For the purpose of the analysis, the interference caused by a BT signal will be approximated based on the effects of a continuous-wave (CW) tone on a DSSS signal [15]. Under this assumption, the interference threshold $\gamma_{1/5}(f_{\text{offset}})$ is

$$\gamma_{1/5}(f_{\text{offset}}) = \gamma_{1/5}(0) - J_S(f_{\text{offset}}) \text{ (dB)}$$

(20)

where

$$J_S(f_{\text{offset}}) = 10\log_{10} \left( \frac{\sin^2(f_{\text{offset}}T_c) G(f_{\text{offset}})^2}{|G(0)|^2} \right) \text{ (dB)},$$

(21)

The $\sin^2(\cdot)$ function results from the spreading of the CW tone at the 802.11b demodulator based on a chip period of $T_c$, and $G(f)$ is the Fourier transform of the chip pulse shaping filter $g(t)$. For the 802.11b with 11 Mbps data rate, $g(t)$ is a fifth-order Butterworth filter with a cutoff frequency of 8.8 MHz [16]. Fig. 6 depicts the graph of the jamming suppression as a function of carrier frequency offset $J_S(f_{\text{offset}})$ based on the CW tone approximation for the BT narrow-band interference.

To facilitate the analysis, a monotonic nonincreasing function was used to estimate $J_S(f_{\text{offset}})$

$$\hat{J}_S(f_{\text{offset}}) = \max_{f \geq f_{\text{off}}} [J_S(f) | f_{\text{off}} - 1 \leq f_{\text{offset}} < f_{\text{off}}]$$

(22)
where
\[ f_q = q F_{\text{step}}; \]
\[ F_{\text{step}} \text{ step size in megahertz for the staircase estimation;} \]
\[ q \in [1 \ldots L]; \]
\[ L = \left\lfloor B_{\text{BT}} / 2F_{\text{step}} \right\rfloor. \]

\( J_S(f_{\text{offset}}) \) is a quantized estimation of \( J_S(f_{\text{offset}}) \) where the argument of the function determines the quantization range \( f_{q-1} \leq f_{\text{offset}} < f_q \) and \( J_S(f_{\text{offset}}) \) is then the maximum value of \( J_S(f) \) evaluated over the range \( f \geq f_{q-1} \). In Fig. 6, \( J_S(f_{\text{offset}}) \) is graphed with \( F_{\text{step}} = 5 \text{ MHz} \). For the coexistence analysis presented in Section IV, \( F_{\text{step}} = 1 \text{ MHz} \) was used. Using (22) with (20) and substituting into (11), a quantized estimate for \( \Gamma \) based on frequency offset is obtained by

\[ \hat{\Gamma}(f_{\text{offset}}) = \gamma_{\text{UL}}(0) - \Omega_{\text{BT}} + \Omega_{\text{AP}} - J_S(f_{\text{offset}}). \]  

The conditional probability of frequency coincidence \( \text{Pr}[C_f | \hat{\Gamma}_q] \) is based on the relative received power from the BT interference to cause \( \hat{\Gamma}_q = \hat{\Gamma}(f_{\text{offset}}) | f_{\text{offset}} < f_q \). To illustrate, for interference to cause performance degradation in the first sidelobe of the 802.11b, the BT signal power has to be at least 28 dB greater than the power required in the 802.11b passband, based on the graph in Fig. 6. If \( \gamma_{\text{UL}}(0) = -10 \text{ dB}, \Omega_{\text{BT}} = 0 \text{ dBm}, \text{ and } \Omega_{\text{AP}} = 20 \text{ dBm}, \) then \( \hat{\Gamma}_q \approx 10 + 2N_{\text{BT}} + 12.5 \text{ MHz} = 38 \text{ dB} \) with \( q = 3 \) and \( F_{\text{step}} = 5 \text{ MHz} \). Since the BT Tx frequency is equilikely within the UL band, the corresponding probability of frequency coincidence is \( \text{Pr}[C_f | \hat{\Gamma}_3] = 2(3 \times 5)/80 = 0.375 \).

Generalizing the example given above, the conditional probability of the frequency overlap given \( \hat{\Gamma}_q \) is

\[ \text{Pr}[C_f | \hat{\Gamma}_q] = \frac{2qF_{\text{step}}}{B_{\text{BT}}}. \]  

This assumes a worst case condition when the 802.11b carrier frequency is centered within the UL band, such that the sidelobe interference can occur for both positive and negative carrier offsets, \( f_{\text{offset}} < f_q \), for the BT interferer.

Based on the method used to obtain \( \hat{\Gamma}(f_{\text{offset}}) \), (22) and (23), the \( L \) events \( \hat{\Gamma}_q \) are mutually exclusive and, as will be shown, \( \sum_{q=1}^{L} \text{Pr}[\hat{\Gamma}_q] \approx 1 \) given that \( F_{\text{step}} \) is sufficiently small. Using the principle of total probability, \( \text{Pr}[C_f] = \sum_{q=1}^{L} \text{Pr}[C_f | \hat{\Gamma}_q] \text{Pr}[\hat{\Gamma}_q] \). The \( \text{Pr}[\hat{\Gamma}_q] \) can be obtained by first noting \( \hat{\Gamma}_q \geq \hat{\Gamma}_{q-1} \forall q \in [1 \ldots L] \). Using (10), the corresponding expected number of BT piconets is also ordered, \( N_{\text{BT}}(\hat{\Gamma}_q) \leq N_{\text{BT}}(\hat{\Gamma}_{q-1}) \). The expected number of BT interferers with \( \hat{\Gamma}_q \) can be estimated by \( N_{\text{BT}}(\hat{\Gamma}_q) - N_{\text{BT}}(\hat{\Gamma}_{q-1}) \). The probability of \( \hat{\Gamma}_q \) is therefore estimated by

\[ \text{Pr}[\hat{\Gamma}_q] = \frac{N_{\text{BT}}(\hat{\Gamma}_{q-1}) - N_{\text{BT}}(\hat{\Gamma}_q)}{N_{\text{BT}}(\hat{\Gamma}_0)} \]
\[ = \frac{A_{\text{eff}}(\hat{\Gamma}_{q-1}, d_S) - A_{\text{eff}}(\hat{\Gamma}_q, d_S)}{A_{\text{eff}}(\hat{\Gamma}_0, d_S)} \]  

where \( \hat{\Gamma}_0 = \hat{\Gamma}(0) = \Gamma(0) \), i.e., \( \hat{\Gamma}(\cdot) \) or \( \Gamma(\cdot) \) evaluated at \( f_{\text{offset}} = 0 \). To illustrate, using the same values for evaluating (23) as in the previous paragraph, and using the results graphed in Fig. 6, for \( q = 0, \ldots, 3 \), \( \approx [10, 13, 38, 40] \). Using the results graphed in Fig. 3 for \( n = 3 \) and \( \sigma_{\text{UL}} = 8 \text{ dB}, \) \( A_{\text{eff}}(\hat{\Gamma}_{q-1}, d_S)/\pi d_S^2 \approx [0.5, 0.27 \times 10^{-3}, 1.5 \times 10^{-3}] \). Using this result with (25), for \( q = 1, \ldots, 3, \) \( \text{Pr}(\hat{\Gamma}_q) \approx [0.6, 0.386, 0.011] \) and \( \sum_{q=1}^{3} \text{Pr}[\hat{\Gamma}_q] \approx \sum_{q=1}^{3} \text{Pr}[\hat{\Gamma}_q] = 0.997 \).
D. Probability of Collision

In this section, a closed-form solution for $\Pr[C]$ is derived based on the results from Sections III-A–C. The probability of collision is first examined based on a single active BT piconet, which is collocated in the same area as the 802.11b STA. This event is denoted as $C_1$ with corresponding probability $\Pr[C_1]$. Based on the probability of frequency coincidence given $\Gamma_q$, (24), and given $n_\tau$, the conditional probability for $C_1$ is

$$\Pr[C_1 | n_\tau, \Gamma_q] = 1 - \left(1 - L_{\text{BT}} \Pr[C_f | \Gamma_q]\right)^{n_\tau}$$  \hspace{1cm} (26)

where BT time slots are equilikely and independently loaded and $L_{\text{BT}}$ is the loading factor for the BT piconet, i.e., the fraction of the total number of BT time slots active. Combining (26) with the results from (19)

$$\Pr[C_1 | n_\tau, \Gamma_q] = \Pr[n_\tau] \Pr[C_1 | n_\tau, \Gamma_q]$$

$$+ \Pr[n_\tau - 1] \Pr[C_1 | n_\tau - 1, \Gamma_q].$$  \hspace{1cm} (27)

Then, using the principle of total probability, the $\Pr[C_1]$ is

$$\Pr[C_1] = \sum_{\tau=1}^{L} \Pr[C_1 | \Gamma_q] \Pr[\Gamma_q].$$  \hspace{1cm} (28)

Equation (28) provides the probability of collision given a single BT piconet is active. Assuming each BT piconet interference...
signal is i.i.d. and letting $C_i$ denote the event, $i$ active BT piconets are collocated in the same region as the 802.11b. Then the corresponding probability is

$$\Pr[C_i] = \sum_{q=1}^{L} \left(1 - \Pr[C_1]^{q} \right)^{q} \Pr[C_i]. \quad (29)$$

The next step in deriving $\Pr[C]$ is combining (29) with the results from Section III-A, i.e., incorporating the expected number of BT piconets, $N_0 \equiv \text{round} \left( N_{\text{BT}} \bar{r}_{0} \right)$, based on the radio propagation parameters and the BT network parameters. Assuming that the probability of activity $\Pr[A_{\text{BT}}]$ is the same for each BT piconet and the activity at each piconet is independent of the activity at other piconets, then

$$\Pr[C] = \sum_{i=1}^{N_0} \left( N_0 \right)^{i} \Pr[A_{\text{BT}}]^{i} \times \left(1 - \Pr[A_{\text{BT}}]\right)^{N_0-i} \Pr[C_i]. \quad (30)$$

### IV. Coexistence Analysis

In this section, the coexistence issue is analyzed between the 802.11b WLAN and BT piconet WPAN. The method used in the analysis was based on using (30) to evaluate $\Pr[C]$. As indicated in the introduction, the goal of the analysis was not to provide an exhaustive examination of the issue of coexistence but to provide insight into the issue and provide a methodology for studying coexistence when there is uncertainty in the network and radio propagation parameters. This is achieved by using the MOP criteria, $U$ and $V$ ([2]), derived in Section II, to relate the $\Pr[C]$ to the MOP performance specifications.

It is first useful to examine (30). There are essentially six independent variables associated with evaluating (30), which can be grouped into two sets of parameters:

1) BT piconet parameters $\equiv [L_{\text{BT}}, D_{\text{BT}}, \Pr[A_{\text{BT}}]]$;
2) radio propagation parameters $\equiv [n, \sigma_{I/S}, d_S]$.

Fixing the radio propagation parameters $[n = 3, \sigma_{I/S} = 8 \text{ dB}, d_S = 15 \text{ m}]$, then $\Pr[C]$ can be evaluated over the BT network parameters. The BT network parameters define a three-dimensional volume. Curves of equal $\Pr[C]$ describe surfaces within the BT parameter space volume, as illustrated in Fig. 7. A surface of equal probability depicted in Fig. 7(a), $\Pr[C] = 0.08$, defines the boundary for $U = [\Pr[C]|\Pr[C] \leq 0.08]$; and in Fig. 7(b), $\Pr[C] = 0.2$ defines the boundary for $V = [\Pr[C]|\Pr[C] \geq 0.2]$.

To put the results depicted in Fig. 7 into perspective, two BT piconet scenarios are considered: $\Delta_L \equiv$ light BT piconet scenario and $\Delta_H \equiv$ heavy BT piconet scenario. A summary of the traffic activity used for $\Delta_L$ and $\Delta_H$ is presented in Table I. The light BT piconet traffic activity was based on estimates for typical usage for a BT piconet in an enterprise setting provided by Bluetooth SIG as reported in [6]. The average call duration of 2 min/call was used for $\Delta_L$, whereas a 1 min/call was suggested in [6]. In addition, BT paging was considered in the analysis presented in [6]. BT paging is used to initiate data or telephony connections. Due to the short time interval spent in the paging mode, paging does not affect the BT piconet parameters, $L_{\text{BT}}$ or $\Pr[A_{\text{BT}}]$, and therefore paging was not incorporated into the analysis presented in this paper. For the heavy BT piconet scenario, two modifications were made in the traffic activity between $\Delta_H$ and $\Delta_L$. First, the number of BT piconets/m$^2$ was doubled. Second, the data transmitted per day were increased from 150 kbytes/day to 20 Mbytes/day in order to model the likelihood that BT piconets would be used to replace wires in a personal computer, i.e., cables to printers, scanners, etc.

Based on the traffic parameters stated in Table I, the BT piconet parameters were obtained; they are given in Table II. The following illustrates the method of obtaining the BT piconet pa-

### TABLE I

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Traffic</th>
<th>BT Packet Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Telephony</td>
<td>10 calls/day @ 2 min/call</td>
<td>HV3</td>
</tr>
<tr>
<td>Email</td>
<td>15 emails/day @ 10 kbytes/email</td>
<td>DH1</td>
</tr>
<tr>
<td>$\Delta_H$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Telephony</td>
<td>10 calls/day @ 2 min/call</td>
<td>HV3</td>
</tr>
<tr>
<td>Email/Printer/Scanner/PC</td>
<td>20 Mbytes/day</td>
<td>Wire Replacement</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$L_{\text{BT}}$</th>
<th>$D_{\text{BT}}$</th>
<th>$\Pr[A_{\text{BT}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_L$</td>
<td>0.3337</td>
<td>0.04</td>
<td>0.042</td>
</tr>
<tr>
<td>$\Delta_H$</td>
<td>0.616</td>
<td>0.08</td>
<td>0.073</td>
</tr>
</tbody>
</table>
rameters based on the traffic parameters for $\Delta_L$. For telephony activity, the time duration per day was 20 min/day. Based on the HV3 packet structure being able to support 386 kbps and a single duplex call requiring 128 kbps, the average loading factor for the telephony activity is 33%. For the data activity, 150 kbytes/day was transmitted using DH1 packet structure. The DH1 packet structure supports 179 kbps throughput with 100% loading. Combining these results for data and telephony, the average values for the $\Delta_L$ scenario, as given in Table II, were obtained.

The two BT piconet scenarios $\Delta_L$ and $\Delta_H$ represent points in the BT piconet parameter space as illustrated in Fig. 7. Using (30), $Pr[C][\Delta_L] \approx 0.07$ and $Pr[C][\Delta_H] \approx 0.34$, where $Pr[C][\Delta_L]$ and $Pr[C][\Delta_H]$ are the $Pr[C]$ evaluated using the BT piconet parameters defined by $\Delta_L$ and $\Delta_H$, respectively. It is clear that $Pr[C][\Delta_L] \in U \Rightarrow 802.11b$ impairment is unlikely for the $\Delta_L$.
scenario and $Pr[C]_{\Delta H} \in V \Rightarrow 802.11b$ impairment is likely for the $\Delta H$ scenario.

Next, the effect of radio propagation parameter variation was investigated by fixing the BT network parameters at both $\Delta L$ and $\Delta H$. Fig. 8(a) and (b) depicts $Pr[C]$ for variations in the radio propagation parameters, as discussed in Section III-A. As can be seen by the results in the graphs, variation in the radio propagation parameters has a significant impact on coexistence. For $\Delta H$, the performance boundary $Pr[C] = 0.2$ was exceeded when $d_S \approx 5$ m for $n = 2$ and $\sigma_{1/S} = 11$ dB. This is due to the relatively large number of BT piconts that can cause interference due to the small path-loss exponent and large variability in the interference to signal power at the STA (Fig. 4). This result can be compared to the nearly four-fold increase in coverage range, $d_S \approx 18$ m, for $n = 2$ and $\sigma_{1/S} = 5$ dB. From the results in Section III-A, for larger path-loss exponents, the effect of $\sigma_{1/S}$ is less significant. This is also consistent with the graphs in Fig. 8. As an example, for $\Delta H$ and for $n = 3$, the boundary $Pr[C] = 0.2$ occurs for $d_S = 8$ to 14 m, and correspondingly, for $n = 4$, the spread in $d_S$ decreases by 50%.

The following method was used to analyze the coexistence issue based on uncertainty in the BT picont parameters. The uncertainty in the BT picont parameters was modeled as an RV $O$. For the analysis presented in this paper, the range for $O$ was defined based on $\Delta L$ and $\Delta H$, where $\Delta L$, defined the lower bound in the parameter space and $\Delta H$ defined the upper bound in the parameter space. The two bounds were used to define a three-dimensional cube in the BT picont parameter space (Fig. 7). The distribution over the range was assumed to be uniform. Using (30), $Pr[U \mid O]$ and $Pr[V \mid O]$ were evaluated numerically where:

1) $Pr[U \mid O]$ is probability BT is unlikely to cause interference based on the specified set of MOPs conditioned on the likely operating region for the BT network parameters;
2) $Pr[V \mid O]$ is probability BT is likely to cause significant interference based on the specified set of MOPs conditioned on the likely operating region for the BT network parameters.

Fig. 9 depicts graphs of $Pr[U \mid O]$ and $Pr[V \mid O]$ versus $d_S$, based on $n = 3$ and $\sigma_{1/S} = 8$ dB. For $d_S < 10$ m, $Pr[U \mid O] > 0.5$ and $Pr[V \mid O] < 0.01$. Therefore, based on the assumptions used in the analysis, it is unlikely the $802.11b$ STA would be affected by the BT piconts for this STA to AP separation. For $d_S > 17$ m, $Pr[U \mid O] \approx 0.01$ and $Pr[V \mid O] > 0.5$; it is, therefore, likely that the $802.11b$ STA would be severely impacted by the BT piconts for this STA to AP separation. Based on the results depicted in Fig. 8, if either $n$ decreases and/or $\sigma_{1/S}$ increases, then the $802.11b$ STA performance is degraded for smaller values of $d_S$.

V. CONCLUSION

WLANs and WPANs coexistence in the UL band is a pressing issue. The methodology used to address this issue centered around deriving a closed-form solution for $Pr[C]$ in terms of the network and radio propagation parameters. In addition, a set of MOPs was derived in terms of $Pr[C]$. In this fashion, the network performance was investigated in regards to the presence of interference. The approach was illustrated based on the coexistence between $802.11b$ and BT UL-band wireless services. Several conclusions concerning the coexistence can be summarized based on the analysis presented.

1) For the BT scenario involving light traffic and a modest picont density, it is unlikely that the $802.11b$ network will be affected for almost any radio propagation environment.
2) For the BT scenario involving moderately heavy traffic and an increase in the picont density, it is likely that the $802.11b$ network will be affected for almost any radio propagation environment.
3) For typical radio propagation values and over a range of BT picont parameters, the $802.11b$ coverage range is likely to be reduced by 50% in order to avoid all or almost all impact from BT interference, and by 15% to avoid severe impact.

The above conclusions were based on specific ranges for radio propagation parameters and BT picont parameters.

The overall methodology presented is applicable to a wide range of network configurations and network performance criteria. The conclusions drawn could be dramatically different, depending on the parameter ranges investigated and the MOPs criteria applied to the analysis.

REFERENCES


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